Constructing Hadamard matrices from orthogonal designs

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Abstract

The Hadamard conjecture is that Hadamard matrices exist for all orders $1, 2, 4t$ where $t \geq 1$ is an integer. We have obtained the following results which strongly support the conjecture:

(i) Given any natural number $q$, there exists an Hadamard matrix of order $2^t q$ for every $s \geq \lceil 2 \log_q (q - 3) \rceil$.

(ii) Given any natural number $q$, there exists a regular symmetric Hadamard matrix with constant diagonal of order $2^s q^3$ for $s$ as before.

A significant step towards proving the Hadamard conjecture would be proving that for any natural number $q$ and constant $c_0$ there exists a Hadamard matrix of order $2^t q$ for some $c < c_0$.

We make steps toward proving the Hadamard conjecture by showing that "If there is an $OD(4p; s_1, s_2, s_3, s_4)$ and a set of $T$-matrices of order $t$ there is an $OD(16p^2 t; 4p s_1, 4p s_2, 4p s_3, 4p s_4)$. In particular, if there is an $OD(4p; p, p, p, p)$ and a set of $T$-matrices of order $t$ there is an $OD(16p^2 t; 4p^2 t, 4p^2 t, 4p^2 t)$. Further, if there are Williamson matrices of order $w$ there is a Hadamard matrix of $16p^2 t w$."
In other words
\[ AA^T = (s_1 x_1^2 + \ldots + s_n x_n^2) f_n. \]

It is known that the maximum number of variables in an orthogonal design is \( \rho(n) \), the Radon number, where for \( n = 2^a b \), \( b \) odd, set \( a = 4c + d \), \( 0 \leq d < 4 \), then \( \rho(n) = 8c + 2^d \).

\( OD(4t; t, t, t) \), otherwise called Bannert-Hall arrays, and \( OD(2^t; a, b, 2^t - a - b) \) have been extensively used to construct Hadamard matrices and weighing matrices. For details see Geramita and Seberry [8].

Cooper and J.S. Wallis (=Seberry) first defined T-matrices of order \( t \) to construct \( OD(4t; t, t, t, t) \) (which at that time they called Hadamard arrays). Four circulant (type 1) matrices \( T_1, T_2, T_3, T_4 \) of order \( t \) which have entries 0, +1 or −1 and which are non-zero for each of the \( t^2 \) entries for exactly one \( i \), i.e.

\[ T_i \ast T_j = 0 \text{ for } i \neq j, \]

where \( \ast \) is the Hadamard (or element by element) product, and which satisfy

\[ \sum_{i=1}^{4} T_i T_i^T = tI, \]

are called T-matrices of order \( t \).

We know that if the row sum (and column sum) of a T-matrix, \( T_i \), of order \( t \) is 0, then

\[ \sum_{i=1}^{4} T_i^2 = t. \]

**Notation.** We use \( T = (t_{ij}) \) given by \( t_{ij} = 1 \) for \( j - i = 1 \) and 0 otherwise for the shift matrix.

Further, we have the following important theorem.

**Theorem 1 (Cooper–Seberry–Turyn).** Suppose there exist T-matrices \( T_1, T_2, T_3, T_4 \) of order \( t \) (assumed to be circulant or block circulant = type \( t \)). Let \( a, b, c, d \) be commuting variables. Then

\[
\begin{align*}
A &= at_1 + bt_3 + ct_5 + dt_4 \\
B &= -bt_1 + at_3 + dt_5 - ct_4 \\
C &= -ct_1 - dt_3 + at_5 + bt_4 \\
D &= -dt_1 + ct_3 - bt_5 + at_4
\end{align*}
\]

can be used in the Goethals-Seidel array (or J. Seberry Wallis-Whiteman array for block-circulant i.e. type 1 and 2 matrices)

\[
\begin{pmatrix}
A & BR & CR & DR \\
-DR & A & DT & -C^T R \\
-C^T R & -DT R & A & B^T R \\
-DR & C^T R & -B^T R & A
\end{pmatrix}
\]

where \( R \) is the permutation matrix which transforms circulant to back-circulant matrices or type 1 to type 2 matrices, to form an \( OD(4t; t, t, t, t) \).
Replacing the variables of Theorem 1 by Williamson type matrices we have:

Method 1 (Cooper–Seberry–Turyn) Suppose there exist $T$-matrices $T_1, T_2, T_3, T_4$ of order $t$ (assumed to be circulant). Let $A, B, C, D$ be Williamson type matrices of order $m$. Then

$$
X = T_1 \times A + T_2 \times B + T_3 \times C + T_4 \times D
Y = T_1 \times -B + T_2 \times A + T_3 \times D + T_4 \times -C
Z = T_1 \times -C + T_2 \times -D + T_3 \times A + T_4 \times B
W = T_1 \times -D + T_2 \times C + T_3 \times -B + T_4 \times A
$$

can be used in the Goethals–Seidel array to form an Hadamard matrix of order $4mt$

$$
GS = \begin{bmatrix}
X & YR & ZR & WR \\
-YR & X & -WT & ZT \\
-ZR & WT & X & -YT \\
-WR & -ZT & YT & X
\end{bmatrix}
$$

Remark 1 The survey of Seberry and Yamamoto [18] gives most presently known T-sequences and $T$-matrices. Some new results are given in this paper. For $t = 67$ there are only $T$-matrices known and not as yet T-sequences. These sequences, using Method 1, are a prolific source of Hadamard matrices and $OD(4t; t, t, t)$.

Turyn has also a construction which says that an $OD(4t; t, t, t, t)$ implies the existence of an $OD(20t; 5t, 5t, 5t, 5t)$ and Ono–Sawade–Yamamoto another which gives an $OD(36t; 9t, 9t, 9t)$ from an $OD(4t; t, t, t, t)$. However neither yields $T$-matrices and neither is recursive. In addition there are $OD(4t; t, t, t, t)$ whenever $2t$ is the order of a Hadamard matrix [14, 6].

Hammer, Sarvate and Seberry [9] applied Kharaghani’s method [11] to $OD(n; s_1, \ldots, s_n)$ and in particular to $OD(4t; t, t, t, t)$ obtaining $OD(12s^2t; 3s^2t, 3s^2t, 3s^2t, 3s^2t)$ and $OD(20s^2t; 5s^2t, 5s^2t, 5s^2t, 5s^2t)$ where $s$ is the length of $T$-sequences.

Yang has other important constructions which give long sequences with zero auto correlation function but not orthogonal designs. There are details in [4]. Yang has given powerful theorems reformulated in [13] which yield many new $OD(4t; t, t, t, t)$ and Hadamard matrices of order $4t$ from $T$-sequences of length $t$. His construction may be stated as

Method 2 If there are base sequences of lengths $m+p, m+p, m, m$ and $y$ is a Yang number then there are $T$-sequences of lengths $t = (2m+p)y$.

For more information on the values $(2m+p)$ and $y$ see [12, 13, 18]. For the known values of Williamson type matrices see [16, 19, 18] and the tables in [10, 18].

We find here the following new orders of Hadamard matrices: $4, q \leq 10, 000$ where $q = 213, 781, 1349, 1491, 1633, 2059, 2627, 2769, 3479, 3763, 4331, 4899, 5467, 5609, 5893, 6177, 6461, 6603, 6887, 7739, 8023, 8591, 9159, 9443, 9727, 9869.$

2 Background and Kharaghani type results

Kharaghani (1985) defined $C_k = |h_{k1}, h_{k2}|$ and applying that to Hadamard matrices of order $4p$ obtained $4p$ symmetric matrices of order $4p$, satisfying

$$
\begin{align*}
C_i C_j & = 0 \\
\sum_{i=1}^{4p} C_i^2 & = (4p)^2 I_{4p}
\end{align*}
$$

\text{ (2) }$
He then used this to show there are Bush-type (blocks $J_{4p}$ down the diagonal) and
Sekekes-type ($h_{ij} = -1 \Rightarrow h_{ji} = 1$ and not necessarily vice versa). By using a symmetric
Latin square he could also have shown that regular symmetric Hadamard matrices with
constant diagonal of order $(4p)^2$ could be constructed by his method.

The result we now give is motivated by Hammer, Sarvate and Seberry [9] but uses a
different technique to obtain more powerful results.

Before proceeding to our main theorem we will illustrate by two examples:

Use Kharaghani’s method to form 4 matrices of order $4p$ satisfying (2) from a Hadamard
matrix of order $4p$.

Use these to form 4 block circulant matrices $A, B, C, D$ with first rows

\[ A : C_1 C_2 \ldots C_{3p} \]
\[ B : C_{3p+1} \ldots C_{4p} C_1 \ldots C_{2p} \]
\[ C : C_{2p+1} \ldots C_{4p} C_1 \ldots C_p \]
\[ D : C_{p+1} \ldots C_{4p} \]

These are now used in a modified Goethals-Seidel or Seberry(Wallis)-Whitman
array. This gives the theorem:

**Theorem 2** If there is a Hadamard matrix of order $4p$ there is a Hadamard matrix of order
$16.3p^2$.

Use Kharaghani’s method to make 4 matrices of order $4p$, $C_1, C_2, \ldots, C_{4p}$ as in Hammer,
Sarvate, Seberry. The matrices now have variable entries

\[ A : C_1, \ldots, C_{p}, C_{2p+1}, C_1, \ldots, C_{4p}, C_{3p} \]
\[ B : C_{3p+1}, \ldots, C_{2p}, C_{2p+1}, C_1, \ldots, C_{4p}, C_{3p} \]
\[ C : C_{2p+1}, \ldots, C_{3p}, C_{1}, C_1, \ldots, C_{2p}, C_{3p} \]
\[ D : C_{p+1}, \ldots, C_{4p}, C_{1}, C_{1}, \ldots, C_{2p}, C_{2p} \]

where $C_{ij} = C_{ji}$.

Use these to form block circulant matrices which are used in the Goethals-Seidel array.
This gives

**Theorem 3** If there is an $OD(4p; p, p, p, p)$ there is an $OD(80p^2; 20p^2, 20p^2, 20p^2, 20p^2)$
and an Hadamard matrix of order $16.5.3p^2$.

These examples do not give new Hadamard matrices of small order but do give new families.
However, if the method is used starting with an $OD(4p; s_1, s_2, s_3, s_4)$ or $OD(4p; p, p, p, p)$
we can get new $OD$’s and Hadamard matrices.

**Theorem 4** Suppose there exists an $OD(4t; s_1, s_2, s_3, s_4)$, in particular an $OD(4t; t, t, t, t)$,
the following $OD$’s exist, the particular case is given in brackets.

(i) $OD(16t^2; 4ts_1, 4ts_2, 4ts_3, 4ts_4)$, ($OD(16t^2; 4t^2, 4t^2, 4t^2, 4t^2)$);
(ii) $OD(48t^2; 12ts_1, 12ts_2, 12ts_3, 12ts_4)$, ($OD(48t^2; 12t^2, 12t^2, 12t^2, 12t^2)$);
(iii) $OD(80t^2; 20ts_1, 20ts_2, 20ts_3, 20ts_4)$, ($OD(80t^2; 20t^2, 20t^2, 20t^2, 20t^2)$).

**Proof.** As in Hammer, Sarvate and Seberry, let $S = (a_{ij})$ be the $OD$. Replace all the
variables of $S$ by 1 making a weighing matrix, $U$, of order $4t$ and weight $w = s_1 + s_2 + s_3 + s_4$
(in the particular case $w = 4p$). Write $S_k$ and $U_k$ for the $k$th rows of $S$ and $U$ respectively. Form

$$C_k = S_k \times U_k^T$$

where $\times$ is the Kronecker product.

Then

$$C_k C_k^T = (S_k \times U_k^T)(S_k \times U_k^T)^T$$

$$= S_k S_k^T \times U_k^T U_k$$

$$= 0$$

if $k \neq j$ because $S$ is an orthogonal design.

Now

$$\sum_{k=1}^{4p} C_k C_k^T = \sum_{k=1}^{4p} (S_k \times U_k^T)(S_k \times U_k^T)^T$$

$$= \sum_{k=1}^{4p} S_k S_k^T \times U_k^T U_k$$

$$= \left( \sum_{j=1}^{4} s_j z_j^2 w \right) I_{4p}$$

by the properties of $U$.

In particular where $s_j = p$, all $j$, we get

$$\sum_{k=1}^{4p} C_k C_k^T = 4p^2(z_1^2 + z_2^2 + z_3^2 + z_4^2)I_{4p}.$$ 

The $C_1, \ldots, C_{4t}$ are now used to form first rows for block circulant matrices, as in the examples leading to Theorems 4 and 5 for (ii) and (iii), or with

$$A : C_1 C_2 \ldots C_t$$

$$B : C_{t+1} \ldots C_{2t}$$

$$C : C_{2t+1} \ldots C_{3t}$$

$$D : C_{3t+1} \ldots C_{4t}$$

for (i).

\[\Box\]

The examples above illustrate that we really need $4t$ matrices $P_1, \ldots, P_{4t}$ of order $q$, with elements 0, +1, −1 such that in each of the $q^2$ places one and only one of the $P_i$ has a nonzero element, i.e. $P_i \ast P_j = 0$, $i \neq j$

$$\sum_{i=1}^{4t} P_i$$ is a $(1, -1)$ matrix

$$P_i P_i^T = \text{constant } I.$$ 

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Then
\[ \sum_{i=1}^{4p} C_i \times P_i \]
would be an Hadamard matrix of order 4tq or an \(OD(4tq; tq, tq, tq, tq)\) say.

Note we need no algebraic relation between the \(P_i\), except disjointness, as \(C_iC_j = 0,\)
\(i \neq j.\)

The remainder of this paper is devoted to finding matrices such as the \(P_i\). We give one
method here which is a blending of ideas derived from writings of Turyn and C.H.Yang.

Let \(h, i, j, k\) be symbols so that \(h^2 = i^2 = j^2 = k^2 = 1, xy = 0, x \neq y, x, y \in \{h, i, j, k\}.\)

Call a sequence of length \(m\) of symbols \(\pm h, \pm i, \pm j, \pm k\) which have zero periodic (or non
periodic) autocorrelation function an \(m, \delta\) sequence.

For example, \(hijj\) is a \(5, \delta\) sequence with zero non-periodic (implies also periodic)
auto-correlation function because for \(hijj\) we form the matrix

\[\begin{bmatrix}
  h & i & i & j & j \\
  0 & h & i & i & j \\
  0 & 0 & h & i & i \\
  0 & 0 & 0 & h & i \\
  0 & 0 & 0 & 0 & h \\
\end{bmatrix}\]

and notice the inner product of any row with any other is zero.

In particular, if \(T_1, T_2, T_3, T_4\) are circulant \(T\)-matrices (which can be obtained from \(T\)-
sequences) of order \(t\) then the first row of

\[X = hT_1 + iT_2 + jT_3 + kT_4\]

is a \(t, \delta\) sequence because

\[XX^T = h^2T_1T_1^T + i^2T_2T_2^T + j^2T_3T_3^T + k^2T_4T_4^T = tI_t\]

using \(xy = 0, x \neq y, x, y \in \{h, i, j, k\}.\)

Construction 1 Suppose we have \(4p\) matrices \(C_1, \ldots, C_{4p}\) of order \(4p\) constructed by Khara-
pauni's method (as modified by Hamner, Sarvate and Seberry (i.e. with variable entries))
and an \(m, \delta\)-sequence \(m_1, m_2, \ldots, m_m.\) To simplify writing write

\[D_h\] for \([C_1 : C_2 : \ldots : C_p]\]
\[D_i\] for \([C_{p+1} : \ldots : C_{2p}]\]
\[D_j\] for \([C_{2p+1} : \ldots : C_{3p}]\]
\[D_k\] for \([C_{3p+1} : \ldots : C_{4p}]\).

We now form 4 first rows of \(A, B, C, D\) by replacing the elements of the \(m, \delta\)-sequence.

To form \(A\) replace \(h\) by \(D_h, -h\) by \(D_{\bar{h}}, \overline{h}\) by \(D_i, i\) by \(D_{\bar{i}}, j\) by \(D_j, j\) by \(-D_j, k\) by \(D_k, -k\) by \(-D_h\) respectively and then complete to a block circulant matrix.

\(A\) is formed by

\[\pm h \rightarrow \pm D_h\]

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\[ \pm i \rightarrow \pm D_i \\
\pm j \rightarrow \pm D_j \\
\pm k \rightarrow \pm D_k \\

B \text{ is formed by} \\
\pm h \rightarrow \pm D_i \\
\pm i \rightarrow \pm D_j \\
\pm j \rightarrow \pm D_h \\
\pm k \rightarrow \pm D_h \\

C \text{ is formed by} \\
\pm h \rightarrow \pm D_j \\
\pm i \rightarrow \pm D_k \\
\pm j \rightarrow \pm D_h \\
\pm k \rightarrow \pm D_i \\

D \text{ is formed by} \\
\pm h \rightarrow \pm D_k \\
\pm i \rightarrow \pm D_h \\
\pm j \rightarrow \pm D_i \\
\pm k \rightarrow \pm D_j \\

Each is then completed to a block circulant matrix.

To illustrate we again use the 5,6-sequence hiijj

\[
A = \begin{bmatrix}
D_h & D_i & D_i & D_j & D_j \\
D_j & D_h & D_i & D_i & D_j \\
D_j & D_j & D_h & D_i & D_i \\
D_i & D_j & D_j & D_h & D_i \\
D_i & D_i & D_j & D_j & D_h 
\end{bmatrix}
\]

where

\[
D_h = \begin{bmatrix}
C_1 & C_2 & C_3 & \cdots & C_p \\
C_p & C_1 & C_2 & \cdots & C_{p-1} \\
\vdots & \ddots & \ddots & \cdots & \vdots \\
C_2 & C_3 & C_4 & \cdots & C_1 
\end{bmatrix}
\]

So

\[
D_h D_h^T = I_p \times \sum_{i=1}^{p} C_i^2
\]

\[D_h D_h^T = 0 \text{ and } D_h D_j^T = 0 \text{ since } C_a C_b = 0, a \neq b.\]

Thus

\[AA^T = I_5 \times (D_h D_h^T + 2D_i D_i^T + 2D_j D_j^T) + (T + T^T) \times (-D_i D_i^T + D_j D_j^T)\]

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Similarly

\[
BB^T = I_5 \times (D_1 D_1^T + 2 D_2 D_2^T + 2 D_3 D_3^T + (T + T^4) \times (-D_1 D_1^T + D_4 D_4^T))
\]

\[
CC^T = I_5 \times (D_1 D_1^T + 2 D_2 D_2^T + 2 D_3 D_3^T + (T + T^4) \times (-D_1 D_1^T + D_5 D_5^T))
\]

\[
DD^T = I_5 \times (D_1 D_1^T + 2 D_2 D_2^T + 2 D_3 D_3^T + (T + T^4) \times (-D_1 D_1^T + D_2 D_2^T))
\]

So

\[
AA^T + BB^T + CC^T + DD^T = 5I_{5p} \times \sum_{i=1}^{4p} C_i^2
\]

\[
= 20p^2(x_1^2 + x_2^2 + x_3^2 + x_4^2)I_{20p^2}
\]

We now use A, B, C, D in the modified GS array to form an OD(80p^2; 20p^2, 20p^2, 20p^2).

Using this method we can establish

**Theorem 5** Suppose an OD(4p; s_1, s_2, s_3, s_4) exists. Suppose there are T-matrices of order t. Then there is an OD(16pt; 4ts_1, 4ts_2, 4ts_3, 4ts_4), an OD(16p^2t; 4pts_1, 4pts_2, 4pts_3, 4pts_4) and an Hadamard matrix of order 16pt and 16p^2t.

**Proof.** The matrix of order 16pt follows by putting the OD(4p; s_1, s_2, s_3, s_4) in place of the variables of the OD(4t; t, t, t, t) constructed via the T-matrices.

The matrix of order 16p^2t is constructed via the construction just given. \(\square\)

**Corollary 1** Suppose an OD(4p; s_1, s_2, s_3, s_4) exists. Then there is an OD(16pt; 4ts_1, 4ts_2, 4ts_3, 4ts_4) and an OD(16p^2t; 4pts_1, 4pts_2, 4pts_3, 4pts_4) for all the orders of T-matrices listed above and in particular for all orders of t \(\leq 100\) except possibly t \(\in\{73, 79, 83, 89, 97\}\).

We give these sequences for odd lengths (Corollary 4.107 is in [7]):

3: \(hij\)

5: \(hhiij\)

7: \(hhhiij\)

9: \(hiiijjjj\)

11: \(hhthithiiij\)

13: Corollary 4.107 \(hhhiijjjjjjj\)

15: \(shykkkhhkkkkkk\)

17: Golay

19: Corollary 4.107 — \(hhhhhiijjjjjkkk\)

21: Golay

23: Corollary 4.107

25: Corollary 4.107

27: Golay

29: Corollary 4.107

31: Corollary 4.107

33: Golay

35: Seberry - Sproul

37: Williamson

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3 New Hadamard matrices

We now give three new T-sequences of lengths $2s + 1 = 35, 61$ and $71$. Each set of sequences is equivalent to a set of base sequences of lengths $s + 1, s + 1, s, s$.

The following are T-sequences (T-matrices) of length $35 = 5^2 + 3^2 + 0^2 + 1^2$.

$$T_1 = \{1, 2, 4, 5, 9, -10, 14, -15, 17\}$$
$$T_2 = \{3, -6, -7, 8, 11, -12, -13, -16, -18\}$$
$$T_3 = \{19, -21, 23, -25, -26, -28, 29, 31, 33, -35\}$$
$$T_4 = \{-20, -22, 24, -27, 30, 32, 34\}$$

The following are T-sequences (T-matrices) of length $61 = 2^2 + 5^2 + 4^2 + 4^2$. Since these sequences are equivalent to base sequences of lengths $31, 31, 30, 30$ they yield, using Yang multipliers, new T-sequences of lengths $183$ and $671$.

$$T_1 = \{1, -2, -4, -6, -8, -10, 12, -14, -16, 18, 20, 22, -24, 26, -28, 30\}$$
$$T_2 = \{3, 5, 7, 9, -11, -13, 15, -17, 19, 21, 23, 25, -27, -29, 31\}$$
$$T_3 = \{-32, -33, -36, 37, 38, 40, -42, 43, 44, 46, -47, 49, 50, 51, 53, -55, -56, 57, -60, 61\}$$
$$T_4 = \{34, -35, 39, 41, -45, 48, -52, 54, 58, 59\}$$

The following are T-sequences (T-matrices) of length $71 = 6^2 + 5^2 + 3^2 + 1^2$.

$$T_1 = \{1, -2, -3, 4, 5, 6, -7, 8, 9, 10, -11, -12, -13, -14, 15, 16, -17, 18, 19, -20, 21, 22, 23, 24\}$$

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\[
\begin{array}{|c|c|}
\hline
q & \text{Method} \\
\hline
213 & 3 \times 71 & 1 \\
781 & 11 \times 71 & 1 \\
1349 & 19 \times 71 & 1 \\
1491 & 21 \times 71 & 1 \\
1633 & 27 \times 71 & 1 \\
2059 & 29 \times 71 & 1 \\
2627 & 37 \times 71 & 1 \\
2769 & 39 \times 71 & 1 \\
3479 & 49 \times 71 & 1 \\
3763 & 53 \times 71 & 1 \\
4331 & 61 \times 71 & 1 \\
4899 & 69 \times 71 & 1 \\
5467 & 7 \times 11 \times 71 & 2 \\
5609 & 79 \times 71 & 1 \\
5893 & 83 \times 71 & 1 \\
6177 & 87 \times 71 & 1 \\
6461 & 91 \times 71 & 1 \\
6603 & 93 \times 71 & 1 \\
6887 & 97 \times 71 & 1 \\
7739 & 71 \times 109 & 1 \\
8023 & 113 \times 71 & 1 \\
8591 & 121 \times 71 & 1 \\
9159 & 129 \times 71 & 1 \\
9443 & 7 \times 19 \times 71 & 2 \\
9727 & 137 \times 71 & 1 \\
9869 & 139 \times 71 & 1 \\
\hline
\end{array}
\]

Table 1 New Hadamard matrices

\[T_2 = \{25, 26, 27, 28, -29, 30, 31, -32, 33, 34, 35, -36, 37, -38, 39, -40, 41, -42, -43, -44, -45, 46, 47\}\]

\[T_3 = \{48, 49, 50, 51, -52, -56, 57, 58, 60, -64, 65, -66, -71\}\]

\[T_4 = \{-53, -54, 55, -59, 61, -62, 63, 67, 68, -69, -70\}\]

The new Hadamard matrices may now be constructed as in Table 1.

Method 3 Seberry and Yamada [18] gave the following definition:

Definition 1 We call \( k \) a Koukovinos–Kounias number, or KK number, if \( k = g_1 + g_2 \) where \( g_1 \) and \( g_2 \) are both the lengths of Golay sequences.

Then we have

Lemma 1 Let \( k \) be a KK number and \( y \) be a Yang number. Then there are \( T \)-sequences of length \( t \) and \( OD(4t; t, t, t) \) for \( t = yk \).
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<td>4</td>
</tr>
<tr>
<td>2227</td>
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</table>

Table 2: New Hadamard matrices of order $2^{s}q$, $t \leq s < t'$

Example. This gives T-sequences of lengths 2.101, 2.109, 2.113, 8.127, 2.129, 2.131, 8.151, 8.157, 16.163, 2.173, 4.179, 4.185, 4.193, 2.201, 2.205, 2.209, 2.213, 2.221, 2.257, 2.261, 2.269.

With the application of this method we find new orders of Hadamard matrices which are given in Table 2.
(Note: $t'$ is given in Jenkins, Koukouvinos and Seberry [10, Table 6].)

References


[10] B. Jenkins, C. Koukouvinos and J. Seberry. Numerical results on T-sequences (odd and even), T-matrices, OD(4t; t, t, t, t), Williamson matrices and Hadamard matrices constructed via OD(4t; t, t, t, t) thence. Technical Report CS88/8, Department of Computer Science, University College, University of New South Wales, ADFA, 1989.


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