Two Level Cretan Matrices Constructed via Singer Difference Sets

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Purpose: This note discusses two level quasi-orthogonal matrices which were first highlighted by J. J. Sylvester; Hadamard matrices, symmetric conference matrices, and weighing matrices are the best known of these matrices with entries from the unit disk. The goal of this note is to develop a theory of such matrices based on preliminary research results. Methods: Extreme solutions (using the determinant) have been established by minimization of the maximum of the absolute values of the elements of the matrices followed by their subsequent classification. Results: We show that if \( B \) is the incidence matrix of a \((\upsilon, k, \lambda)\) difference set, then there exists a two-level quasi-orthogonal matrix, \( S \), a Cretan(\( \upsilon \)) matrix. We apply this result to the Singer family of difference sets obtaining a new infinite family of Cretan matrices. Practical relevance: Web addresses are given for other illustrations and other matrices with similar properties. Algorithms to construct Cretan matrices have been implemented in developing software of the research program-complex.

Keywords – Hadamard matrices; quasi-orthogonal matrices; Cretan matrices; difference sets; Singer difference sets; Hadamard difference sets.

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