MATH312 Tutorial Questions
Autumn 2008
Week 5

Question 1:

1. Let $\underset{\sim}{A}$ be a vector, which can be represented in Cartesian coordinates as
$$\underset{\sim}{A} = 2\underset{\sim}{e}_1 - \underset{\sim}{e}_2 + 3\underset{\sim}{e}_3.$$
   (a) If $\underset{\sim}{A}$ starts at the origin, sketch the geometrical representation of $\underset{\sim}{A}$.
   (b) Describe in words the *movement* of $\underset{\sim}{A}$, from where it starts to where it finishes.
   (c) Without using any formulae, write down the physical components of $\underset{\sim}{A}$.

2. Consider vector $\underset{\sim}{B}$, which can be expressed as
$$\underset{\sim}{B} = 3\underset{\sim}{E}_1 + \underset{\sim}{E}_2 - 3\underset{\sim}{E}_3,$$
where $(\underset{\sim}{E}_1, \underset{\sim}{E}_2, \underset{\sim}{E}_3)$ are a set of tangential basis vectors in directions of a orthogonal curvilinear coordinate system (X^1, X^2, X^3).
   (a) If $\underset{\sim}{B}$ starts at the origin, sketch the geometrical representation of $\underset{\sim}{B}$.
   (b) Describe in words the *movement* of $\underset{\sim}{B}$, from where it starts to where it finishes.
   (c) Determine the physical components of $\underset{\sim}{B}$.

3. Let $\underset{\sim}{C}$ be a vector that can be expressed as
$$\underset{\sim}{C} = C_1\underset{\sim}{E}^1 + C_2\underset{\sim}{E}^2 + C_3\underset{\sim}{E}^3,$$
where $(\underset{\sim}{E}^1, \underset{\sim}{E}^2, \underset{\sim}{E}^3)$ are a set of gradient basis vectors of the orthogonal curvilinear coordinate system (X^1, X^2, X^3).
   (a) Determine the general relationship between the tangential basis vectors $(\underset{\sim}{E}_1, \underset{\sim}{E}_2, \underset{\sim}{E}_3)$ and the gradient basis vectors $(\underset{\sim}{E}^1, \underset{\sim}{E}^2, \underset{\sim}{E}^3)$ for this orthogonal curvilinear coordinate system (X^1, X^2, X^3).
   (b) If $\underset{\sim}{C}$ starts at the origin, sketch the geometrical representation of $\underset{\sim}{C}$.
   (c) Describe in words the *movement* of $\underset{\sim}{C}$, from where it starts to where it finishes.
   (d) Determine the physical components of $\underset{\sim}{C}$.

4. Let (X^1, X^2, X^3) refer to the modified cylindrical polar coordinates, which have the transformation relations
$$z^1 = 3X^1 \cos X^2, \qquad z^2 = 3X^1 \sin X^2, \qquad z^3 = 2X^3.$$
   (a) Find the tangential basis vectors $(\underset{\sim}{E}_1, \underset{\sim}{E}_2, \underset{\sim}{E}_3)$ and the gradient basis vectors $(\underset{\sim}{E}^1, \underset{\sim}{E}^2, \underset{\sim}{E}^3)$.
   (b) Find the physical components of $\underset{\sim}{B}$.
   (c) Find the physical components of $\underset{\sim}{C}$.

please turn over for some more fun...

Question 2:

Let t^i_j be a second-order mixed tensor given by

$$t^i_j = \begin{pmatrix} 1 & a(\underset{\sim}{X}) & b(\underset{\sim}{X}) \\ c(\underset{\sim}{X}) & 1 & 0 \\ d(\underset{\sim}{X}) & 0 & e(\underset{\sim}{X}) \end{pmatrix},$$

where $a(\underset{\sim}{X})$, $b(\underset{\sim}{X})$, $c(\underset{\sim}{X})$, $d(\underset{\sim}{X})$ and $e(\underset{\sim}{X})$ are scalar functions of $\underset{\sim}{X} = (X^1, X^2, X^3)$. Find the physical components of t^i_j, if

1. (X^1, X^2, X^3) refer to the usual Cartesian coordinates,
2. (X^1, X^2, X^3) refer to the usual cylindrical polar coordinates,
3. (X^1, X^2, X^3) refer to the usual spherical polar coordinates,
4. (X^1, X^2, X^3) refer to the modified cylindrical polar coordinates given in QuestionNumber 1:4(d).

For each of the physical tensors found above, what is implicity implied about the dimensions of each element, and hence, give an example of a valid functional form for each scalar function.

Question 3:

Let (x^1, x^2, x^3) denote the usual Cartesian coordinate system and (X^1, X^2, X^3) denote an orthogonal curvilinear coordinate system, where both are centered at the same origin O. Find the direction cosines if:

1. OX^1 and OX^2 are simply rotated counter-clockwise by $30°$ in the $x^1 - x^2$ plane, while OX^3 is in the same direction as Ox^3.
2. OX^1 and OX^2 are simply rotated clockwise by $15°$ in the $x^1 - x^2$ plane, while OX^3 is in the same direction as Ox^3.
3. OX^1 and OX^3 are simply rotated counter-clockwise by $45°$ in the $x^1 - x^3$ plane, while OX^2 is in the same direction as Ox^2.

Sketch each situation described above.