MATH312 Tutorial Questions
Autumn 2008
Week 4

Question 1:

Consider the modified cylindrical polar coordinates (r, θ, z) where

$$x = r\cos\theta, \qquad y = r\sin\theta - \theta, \qquad z = z.$$

Given that

$$g_{ij} = \frac{\partial z^k}{\partial X^i}\frac{\partial z^k}{\partial X^j}, \qquad g^{jk} = \frac{(-1)^{j+k}G_{jk}}{g},$$

then find:

1. The metric tensor g_{ij}.
2. The conjugate metric tensor g^{jk}.
3. The Jacobian determinant J.

Question 2:

Write down and classify the appropriate transformation laws for the following tensors.

1. A_{ij}
2. B^i_j
3. C
4. ε_{ijk}
5. I^{hi}_{bye}

Question 3:

Consider the prolate spheroidal coordinates (u, v, ϕ)

$$x = a\sinh u \sin v \cos\phi, \qquad x = a\sinh u \sin v \sin\phi, \qquad z = a\cosh u \cos v.$$

Find:

1. The position vector $\underset{\sim}{r} = \underset{\sim}{r}(u, v, \phi)$.
2. Find the set of tangential base vectors $(\underset{\sim}{E}_1, \underset{\sim}{E}_2, \underset{\sim}{E}_3)$.
3. Determine the metric tensor g_{ij} using the fact that

$$g_{ij} = \underset{\sim}{E}_i \cdot \underset{\sim}{E}_j.$$

please turn over for some more fun. . .

Question 4:

For the following tensors, raise all subscript indices and lower all superscript indices

1. T_j
2. H_i^{ts}
3. D^j_{abcde}
4. g^{jk}
5. $\dfrac{\partial x^i}{\partial X^j}$

Question 5:

Consider the coordinate transformation from Cartesian coordinates (z^1, z^2, z^3) into cylindrical polar coordinates (X^1, X^2, X^3), namely

$$z^1 = X^1 \cos X^2, \qquad z^2 = X^1 \sin X^2, \qquad z^3 = X^3,$$

and let the vector $\underset{\sim}{A}$ be represented by

$$\underset{\sim}{A} = A_x \underset{\sim}{e}_1 + A_y \underset{\sim}{e}_2 + A_z \underset{\sim}{e}_3.$$

Then:

1. Sketch the geometrical representation of the coordinate transformation.
2. Find the inverse transformation.
3. Show that the set tangential basis vectors $(\underset{\sim}{E}_1, \underset{\sim}{E}_2, \underset{\sim}{E}_3)$ are related to the usual Cartesian orthonormal basis vectors $(\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3)$ by
$$\underset{\sim}{E}_1 = \cos X^2 \underset{\sim}{e}_1 + \sin X^2 \underset{\sim}{e}_2, \qquad \underset{\sim}{E}_2 = -\sin X^2 \underset{\sim}{e}_1 + \cos X^2 \underset{\sim}{e}_2, \qquad \underset{\sim}{E}_3 = \underset{\sim}{e}_3.$$
4. Show that the tangential basis vectors given above in 3. are unit vectors.
5. Find the contravariant physical components $A^{(1)}$, $A^{(2)}$ and $A^{(3)}$ of $\underset{\sim}{A}$.
6. Determine the gradient basis vectors $(\underset{\sim}{E}^1, \underset{\sim}{E}^2, \underset{\sim}{E}^3)$.
7. Find the covariant physical components $A_{(1)}$, $A_{(2)}$ and $A_{(3)}$ of $\underset{\sim}{A}$.