

MATH312: Surprise Test – Week 3

Question 1:

Show that

1. $\delta_{ij} \delta_{ij} = 3$
2. $\varepsilon_{ijk} \varepsilon_{jki} = 6$
3. $\delta_{ij} \varepsilon_{ijk} = 0$

Question 2:

If \underline{f} , \underline{g} and \underline{h} are vectors, then express the following in tensorial notation.

1. $\underline{g} \cdot \underline{h}$
2. $\underline{h} \times \underline{g} \cdot \underline{f}$
3. $\underline{g} \times \underline{f} \cdot \underline{h}$
4. $\underline{\nabla} \cdot \underline{f} \times \underline{g}$

Question 3:

Let $\Phi = \Phi(x^1, x^2, x^3)$. Consider the coordinate transformation

$$x^1 = X^1 \cos X^2, \quad x^2 = X^1 \sin X^2, \quad x^3 = X^3.$$

Find

1. The Jacobian J
2. $\frac{\partial \Phi}{\partial X^2}$
3. the metric tensor components g_{11} and g_{22}
4. the determinant of the metric tensor, g
5. the metric tensor component g_{33} , but without using the formula for the metric tensor, given the fact that here the metric tensor is a diagonal matrix

Question 4:

Consider the coordinate transformation

$$x = r \cos \theta, \quad y = \theta + r \sin \theta, \quad z = z.$$

Find the metric tensor g_{ij} and the conjugate metric tensor g^{jk} .

please turn over for some more fun...

Question 5:

Give an example of the following quantities, and if possible the corresponding transformation laws.

1. A scalar
2. A covariant tensor
3. A contravariant tensor
4. A covariant tensor of order 3
5. A mixed tensor

Question 6:

Ronald Ralf, a reporter for MAS_{NEWS}, went walking alone in the dark woods one night, when he stumbled across a clearing at the branch between two rivers. On the ground, at his feet, he saw the following inscription:

“All things rubbery and stretchy shall be ruled by

$$(\lambda + \mu)\nabla(\nabla \cdot \underline{u}) + \mu\nabla^2 \underline{u} = 0$$

So say Me the Wise.”

Help Ronald understand this dark message by

1. expressing the equation in tensorial form - assuming Cartesian coordinates
2. suggesting a title for Ronald's story about this equation