MATH312 Tutorial Questions
Autumn 2008
Week 2

Question 1:

For the stated tensorial operations, determine if valid, and if so, what is the resulting tensor.

1. $A^i + B^i$
2. $A^i B_j$
3. $A^i B^m_{jk} + D^{im}_j E_k$
4. $\delta_{ij} + \varepsilon_{ijk}$

Question 2:

If σ_{ij} is the second order tensor defined by

$$\sigma_{ij} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix},$$

then calculate

1. σ_{ii},
2. $\sigma_{ij}\,\sigma_{ij}$,
3. $\sigma_{1j}\,\sigma_{j2}$.

Question 3:

Show that

1. $\delta_{ii} = 3$,
2. $\delta_{ij}\,\delta_{ij} = 3$,
3. $\varepsilon_{ijk}\,\varepsilon_{jki} = 6$,
4. $\varepsilon_{ijk}\,A_j A_k = 0$,
5. $\delta_{ij}\,\delta_{jk} = \delta_{ik}$,
6. $\delta_{ij}\,\varepsilon_{ijk} = 0$.

please turn over for some more fun...

Question 4:

Ronald Ralf, a reporter for MAS_{news}, is investigating the mysterious appearance of a pattern in the local wheat fields. Viewing the pattern from above in Doc Chopper's helicopter, Ronald sees the equation

$$G\left(u_{i,kk} + \frac{1}{1-2\nu}u_{k,ki}\right) + X_i = \rho\frac{\partial^2 u_i}{\partial t^2}.$$

Trying to decipher the inscription, Ronald goes to a library and from his exhaustive research, he has been able to piece together that G, ν and ρ are constants, but can go no further. To help Ronald:

1. Identify the free and dummy indices, stating their ranges.
2. Explain the summation convention.
3. Write out the equation in expanded form.
4. Suggest a title for Ronald's article.

Question 5:

Sally Forth, Ronald Ralf's niece, has been asked to express the following equations as a single equation using index notation.

$$\begin{aligned}
\epsilon_{xx} &= \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right], \\
\epsilon_{yy} &= \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right], \\
\epsilon_{zz} &= \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right], \\
\epsilon_{xy} &= \frac{1+\nu}{E}\sigma_{xy}, \\
\epsilon_{yz} &= \frac{1+\nu}{E}\sigma_{yz}, \\
\epsilon_{xz} &= \frac{1+\nu}{E}\sigma_{xz}.
\end{aligned}$$

If Sally has also been told that $\epsilon_{ij} = \epsilon_{ji}$ and $\sigma_{ij} = \sigma_{ji}$, for $i, j = x, y, z$, then rewrite the equations as a single equation (Hint: make use of the Kronecker delta).