Aggregation of Seasonally Adjusted Estimates by a Post-Adjustment

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Chapter 1

Introduction

Time series can be calculated as the sum of a number of time series. For example, a national total could equal the sum of state totals or the sum of industry totals. In each case the national total is referred to as the aggregated series. Depending on how the aggregated series is derived the state totals or industry totals are referred to as the component series. This is an example of aggregation in one dimension. This aggregation could occur in two dimensions. For example, estimates for industries within each state which sum to marginal state totals, marginal industry totals and also to a national total over all industries.

Assume that the original series can be decomposed using a multiplicative relationship. This can be represented as

$$\begin{aligned} O_t &= T_t \times S_t \times I_t \\ SA_t &= \frac{O_t}{\hat{S}_t} \end{aligned}$$

where O_t, T_t, S_t and I_t refer to the original estimates, trend, seasonal and irregular components respectively at time t. $\hat{S}_t$ is an estimate of the seasonal component. SA_t refers to the seasonally adjusted estimate at time t. Users of official government statistics tend to focus on the seasonally adjusted estimates and also the trend which can be derived from the seasonally adjusted estimates.

There is demand from users of Australian Bureau of Statistics (ABS) data that the additive relationship between the original component and aggregate series be reflected in the seasonally adjusted component and aggregate series. Direct seasonal adjustment by the X-11 method (Shiskin *et al.*, 1967) does not preserve the additive relationship that exists between the aggregated original series and the component original series.

That is, for k component time series, $\sum_{h=1}^{k} O_t^{(h)} = O^{(\text{Total})}$ whereas $\sum_{h=1}^{k} SA_t^{(h)} \neq SA^{(\text{Total\_direct})}$ where the superscript (h) indexes over the k

component series and $SA^{(\text{Total\_direct})}$ is the directly seasonally adjusted aggregated series.

Most government statistical agencies, including the ABS for some cases, use a post-adjustment correction to ensure additive that relationships in the seasonally adjusted estimates are preserved. For example, by applying weights $w_t^{(h)}$ to each of the component series such that

$$\sum_{h=1}^{k} w_t^{(h)} SA_t^{(h)} = SA_t^{(\text{Total\_direct})} \tag{1.1}$$

In the ABS, direct seasonal adjustment of aggregated series and a post-adjustment correction is currently used in a number of situations, including the systems of series Building Activity (ABS, 2004a) and Retail Trade (ABS, 2004b).

The aim of this paper is to identify a method of post-adjustment correction that is an improvement on the current methodology used in the ABS and is practical to implement in an ABS environment. Chapter 2 discusses methods that incorporate the level, movement and volatility of the series and the current ABS practice of proration. Chapter 3 presents the mathematical details of the methods studied. Extensions to these methods are also presented that draw together some of the competing objectives of a post-adjustment. Chapter 4 describes the simulation study undertaken to evaluate the performance of the methods proposed. Chapter 4 includes the results of the study. Chapter 5 poses questions for the Methodological Advisory Committee (MAC) to help guide directions for future study.

Chapter 2

Literature Review

The problem of ensuring aggregation in a system of time series is commonly referred to in the literature as benchmarking to contemporaneous constraints. This is as distinct from benchmarking to temporal constraints. Benchmarking to contemporaneous constraints involves forcing consistency across a system of series, one time point at a time. Benchmarking to temporal constraints involves enforcing consistency across time with respect to another time series.

Much of the early work on benchmarking relates to benchmarking to temporal constraints. In particular, forcing time series estimates to be consistent with estimates from a time series of lower frequency. For example, forcing quarterly estimates to add to annual totals. Such methods relate well to the problem of benchmarking to contemporaneous (cross-sectional) constraints, however, caution must be used when preservation of movement estimates is the objective (Cholette and Chhab, 2004).

With the exception of Den Butter and Fase (1991), the reviewed literature does not specifically account for the problem of non-additivity in a system of seasonally adjusted time series. This non-additivity is casued by the non- linearity of the seasonal adjustment process. The added information available about the seasonal adjustment process that caused the loss of additivity can be used when choosing a correction method. For example, a time series analyst may be less willing to make large changes to a series that can be seasonally adjusted with confidence.

2.1 Denton Method (1971)

Denton (1971) describes a method for implementing benchmarking to temporal constraints while minimising the changes to the period to period movement of the estimates. This is now referred to as the Denton principle.

That is, minimising the sum

$$\sum_t \sum_{h=1}^{k} \left(\left(SA_t^{(h)} - SA_{t-1}^{(h)} \right) - \left(w_t^{(h)} SA_t^{(h)} - w_{t-1}^{(h)} SA_{t-1}^{(h)} \right) \right)^2 \qquad (2.1)$$

Equation (2.1) can be written succinctly in matrix form (see Section 3.2). The complex matrix algebra required to solve the minimisation can be simplified by taking into account the structure of the matrices involved. The time series estimates in question do not necessarily have to be seasonally adjusted estimates but can be any time series estimates that need to be aggregated to lower frequency estimates. The concept of preservation of movements is one that can be incorporated into the ABS problem of aggregation to contemporaneous constraints.

2.2 den Butter and Fase (1990)

Den Butter and Fase (1990) describe methods used in the Nederlanchsche Bank for the contemporaneous aggregation for seasonally adjusted estimates. This method uses constrained minimisation of the squared differences between the initial seasonally adjusted and the corrected seasonally adjusted estimates where the summand for each series is weighted by the inverse of the product of the standard errors of the irregular and seasonal components of the series. That is, setting the weights in (1.1) to minimise

$$\sum_t \sum_{h=1}^{k} \frac{1}{\sigma_I^{(h)} \sigma_S^{(h)}} \left(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)} \right)^2$$

where $\sigma_I^{(h)}$ and $\sigma_S^{(h)}$ are the standard errors of the irregular and seasonal components respectively for series h.

This method relates specifically to the problem of benchmarking seasonally adjusted estimates produced by the X-11 program (Shiskin *et al.*, 1967) to contemporaneous constraints. In the multiplicative seasonal decomposition case, this method does not account for the level of the series as the irregular and seasonal components are independent of the level. That is, if two series of very different levels have the same standard errors for the seasonal and irregular components, they will be corrected by the amount in an additive sense. In proportional terms, this means the lower level series will have a greater correction.

2.3 Di Fonzo and Marini (2003)

Di Fonzo and Marini (2003) describe simultaneous benchmarking to temporal and contemporaneous constraints using the Denton principle of movement preservation. They focus on the aggregation of seasonally adjusted series and note that

> "The proportional adjustment of a system of time series mostly alters those component series having greater magnitude. ...the ranges of corrections present (quasi) perfect correlation with the ranking (by means) of the variables. Such a result might be in contrast with the fact that the most reliable series of a survey are generally the greater ones (and *viseversa*)." p17

This observation is because the volatility in the seasonal and irregular component are not explicitly entered into the minimisation problem. In general, higher level series have lower volatility in a multiplicative sense and that is not accounted for in this framework.

2.4 Cholette and Chabb (2004)

Cholette and Chabb (2004) have developed a Generalised Benchmarking System (GBS) used within Statistics Canada. This system meets both temporal and contemporaneous constraints of the time series. As in Denton (1971), the estimates in used are not necessarily seasonally adjusted estimates. The volatility of the time series can be taken into account by using the covariance matrices of the time series or the Denton principle of movement preservation (or even a relaxed version of this constraint). The volatility is in a general time series sense and does not account for the particular case of seasonal adjustment. Cholette and Chabb (2004) state that the Denton principle can produce large changes to the level of the series if temporal constraints are not present. The GBS allows for aggregation in two contemporaneous dimensions. The matrix algebra for the two dimensional case is a relatively simple extension of that of the one dimensional case.

2.5 Current ABS Methodology of Proration

The ABS currently performs seasonal adjustment using SEASABS (seasonal analysis ABS standards) (ABS, 2001). SEASABS is an enhanced version of the X-11 methodology (Shiskin *et al.*, 1967) and includes a link to X-12-ARIMA (Findley *et al.*, 1998). SEASABS can be used to document the relationships that exist between a system of time series. This is achieved by using an aggregation structure. An aggregation structure can include specifications about which aggregate series are to be directly seasonally adjusted and which are to be indirectly seasonally adjusted.

In practice, a post-adjustment correction can be used to ensure additivity within an aggregation structure of the seasonally adjusted series as follows

1. the directly seasonally adjusted aggregated series is considered fixed and has no further adjustments made, and

2. the directly seasonally adjusted component series are weighted to ensure additivity of the seasonally adjusted series.

The current method of aggregation used in the ABS sets $w_t^{(h)}$ in (1.1) to be equal for all $h = 1, 2, \ldots, k$. The term proration is used to describe this special case of aggregation. The result of this weighting method is that the size of the post-adjustment to each component series is proportional to the level of the series, i.e.

$$\begin{aligned} w_t^{(h)} &= \frac{SA^{(\text{Total\_direct})}}{\sum_{h=1}^{k} SA_t^{(h)}} \\ &= \frac{SA^{(\text{Total\_direct})}}{SA^{(\text{Total\_indirect})}} \\ \Rightarrow SA_t^{(h)} - w_t^{(h)} SA_t^{(h)} &= SA_t^{(h)} - \frac{SA^{(\text{Total\_direct})}}{SA^{(\text{Total\_indirect})}} SA_t^{(h)} \\ &= SA_t^{(h)} \left(1 - \frac{SA^{(\text{Total\_direct})}}{SA^{(\text{Total\_indirect})}} \right) \\ &\propto SA_t^{(h)} \end{aligned} \tag{2.2}$$

The proration approach is simple to implement and has the property that the larger level series undergo a larger absolute correction. Possible limitations to the proration approach are that it:

a) can result in a change of the direction of a movement in seasonally adjusted terms;

b) takes no account of the relative volatility of the component series; and

c) may result in larger corrections than are necessary to restore additivity in the seasonally adjusted estimates.

Chapter 3

Quality measures to assess a post adjustment correction

This study uses a range of quality measures to assess each post-adjustment correction. In this study we have focused on

1. that the post-adjustment results in a small correction to the level of the seasonally adjusted series (Section 3.1);

2. that the post-adjustment results in a small correction to the period to period movements of the seasonally adjusted series (Section 3.2);

3. that highly volatile series are altered more than less volatile series (Section 3.3).

Combinations of these different measures are also considered in Sections 3.4-3.7. Alternative quality measures not considered in this case are

4. that there is no introduction of residual seasonality into the seasonally adjusted series;

5. ensuring consistency between component series in relation to prior corrections such as Easter proximity, trading day effects and corrections for large extremes.

Consider a general form of an objective function that can be adapted to measure the quality measure in points 1-3. This general form will be shown to be suitable for the combinations of the three quality measures. The objective function given by

$$(\widetilde{\mathbf{sa}} - \mathbf{sa})'\mathbf{\Pi}^{\frac{1}{2}}\mathbf{\Lambda}\mathbf{\Pi}^{\frac{1}{2}}(\widetilde{\mathbf{sa}} - \mathbf{sa}) \qquad (3.1)$$

subject to

$$\mathbf{B}\widetilde{\mathbf{sa}} = \mathbf{sa}^{(Total\_direct)}$$

where

$$
\begin{aligned}
\mathbf{sa} &= \begin{bmatrix} SA_1^{(1)} \\ \vdots \\ SA_n^{(1)} \\ \cdots \\ \vdots \\ \cdots \\ SA_1^{(k)} \\ \vdots \\ SA_n^{(k)} \end{bmatrix}_{nk\times 1}, \quad \widetilde{\mathbf{sa}} = \begin{bmatrix} w_1^{(1)} SA_1^{(1)} \\ \vdots \\ w_n^{(1)} SA_n^{(1)} \\ \cdots \\ \vdots \\ \cdots \\ w_1^{(k)} SA_1^{(k)} \\ \vdots \\ w_n^{(k)} SA_n^{(k)} \end{bmatrix}_{nk\times 1}, \\
\mathbf{sa}^{(Total\_direct)} &= \begin{bmatrix} SA_1^{(Total\_direct)} \\ \vdots \\ SA_n^{(Total\_direct)} \end{bmatrix}_{n\times 1} \\
\mathbf{B}' &= [I_n, I_n, \ldots, I_n]_{n\times nk} \\
\mathbf{\Pi} &= \text{diag}\left[\pi_1^{(1)}, \pi_2^{(1)}, \ldots, \pi_n^{(1)}, \ldots, \pi_1^{(k)}, \pi_2^{(k)}, \ldots, \pi_n^{(k)}\right] \\
n &= \text{number of data points of each time series} \\
&\quad \text{(length of component time series could differ but we consider} \\
&\quad \text{them equal here for simplicity)} \\
k &= \text{number of component time series in the system.}
\end{aligned}
$$

Different objective functions are defined by different choices of the matrices $\mathbf{\Lambda}$ and $\mathbf{\Pi}$. The matrix $\mathbf{\Lambda}$ defines whether the objective function should deal with changes to movement or to level. The matrix $\mathbf{\Pi}$ defines whether the objective function should measure absolute or porportional changes to level or movement. The matrix $\mathbf{\Pi}$ can incorporate a penalty that ensures highly volatile time series are altered by more than less volatile time series. Different objective functions are defined by different choices of the matrices $\mathbf{\Pi}$ and $\mathbf{\Lambda}$.

Minimising the objective function in (3.1) with respect to $\widetilde{\mathbf{sa}}$ leads to the general solution

$$\widetilde{\mathbf{sa}} = \mathbf{sa} + \mathbf{\Pi}^{-\frac{1}{2}}\,\mathbf{\Lambda}^{-1}\,\mathbf{\Pi}^{-\frac{1}{2}}\mathbf{B}(\mathbf{B}'\mathbf{\Pi}^{-\frac{1}{2}}\,\mathbf{\Lambda}^{-1}\mathbf{\Pi}^{-\frac{1}{2}}\,\mathbf{B})^{-1}[\mathbf{sa}^{(Total\_direct)} - \mathbf{B}'\,\mathbf{sa}] \tag{3.2}$$

Sections 3.1-3.7 use the general form of (3.1) and, in doing so, allow the use of the solution (3.2).

3.1 Changes to Level of Uncorrected Series

It seems intuitively reasonable that the post adjustment corrected seasonally adjusted estimates of the component series should closely resemble the initial seasonally adjusted estimates. This can be measured in terms of the absolute change or proportional change, squared absolute change or squared proportional change between the corrected and uncorrected seasonally adjusted estimates. These considerations lead to defining the following measures of deviation as possible quality measures for the post-adjustment

$$\sum_t \sum_{h=1}^{k} \big(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}\big)^2 \tag{3.3}$$

$$\sum_t \sum_{h=1}^{k} \frac{\big(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}\big)^2}{SA_t^{(h)}} \tag{3.4}$$

$$\sum_t \sum_{h=1}^{k} \Big(\frac{SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}}{SA_t^{(h)}}\Big)^2 \tag{3.5}$$

Minimising (3.4) leads to the proration approach described in Section 2.5.

The three objective functions in (3.3), (3.4) and (3.5) are special cases of the general expression (3.1) where $\mathbf{\Lambda}$ is the identity matrix and the elements of the matrix $\mathbf{\Pi}$ are defined as

$$\pi_t^{(h)} = 1 \tag{3.6}$$

$$\pi_t^{(h)} = \frac{1}{SA_t^{(h)}} \tag{3.7}$$

$$\text{and} \quad \pi_t^{(h)} = \frac{1}{\big(SA_t^{(h)}\big)^2} \tag{3.8}$$

respectively.

3.2 Changes to Absolute and Percentage Movements of Uncorrected Series

Users of time series data are often more concerned with the period to period movement of the seasonally adjusted estimates than with the level of the seasonally adjusted estimates. Users may have less confidence in the post- adjustment process if the direction of movements are not preserved after the post adjustment correction has been applied.

The method proposed by Denton (1971) minimises the sum of the squared movements or the sum of the squared proportional movements. That is, minimising the objective functions

$$
\begin{aligned}
& \sum_t \sum_{h=1}^{k} \Big(\big(SA_t^{(h)} - SA_{t-1}^{(h)}\big) - \big(w_t^{(h)} SA_t^{(h)} - w_{t-1}^{(h)} SA_{t-1}^{(h)}\big) \Big)^2 \\
= & \sum_t \sum_{h=1}^{k} \Big(\big(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}\big) - \big(SA_{t-1}^{(h)} - w_{t-1}^{(h)} SA_{t-1}^{(h)}\big) \Big)^2 \qquad (3.9)
\end{aligned}
$$

and also

$$
\begin{aligned}
& \sum_t \sum_{h=1}^{k} \left(\frac{\big(w_t^{(h)} SA_t^{(h)} - SA_t^{(h)}\big)}{SA_t^{(h)}} - \frac{\big(w_{t-1}^{(h)} SA_{t-1}^{(h)} - SA_{t-1}^{(h)}\big)}{SA_{t-1}^{(h)}} \right)^2 \\
= & \sum_t \sum_{h=1}^{k} \Big(w_t^{(h)} - w_{t-1}^{(h)} \Big)^2 \qquad (3.10)
\end{aligned}
$$

The minimisation of the objective function in (3.10) minimises the sum of the squared changes in proportional correction. Di Fonzo and Marini (2003) consider this a good approximation to the more preferable form of

$$
\begin{aligned}
& \sum_t \sum_{h=1}^{k} \left(\frac{\big(w_t^{(h)} SA_t^{(h)} - w_{t-1}^{(h)} SA_{t-1}^{(h)}\big)}{w_{t-1}^{(h)} SA_{t-1}^{(h)}} - \frac{\big(SA_t^{(h)} - SA_{t-1}^{(h)}\big)}{SA_{t-1}^{(h)}} \right)^2 \\
= & \sum_t \sum_{h=1}^{k} \left(\frac{SA_t^{(h)}}{SA_{t-1}^{(h)}} \Big(\frac{w_t^{(h)}}{w_{t-1}^{(h)}} - 1 \Big) \right)^2 \qquad (3.11)
\end{aligned}
$$

which is nonlinear and requires numeric solution methods (Fagan, 1995).

The objective functions in (3.9) and (3.10) can be expressed in matrix form by introducing a differencing matrix $\mathbf{D}$ and setting $\mathbf{\Lambda}$ in (3.1) as

$$\mathbf{\Lambda} = \mathbf{D}'\mathbf{D}$$

Denton (1971) defines the matrix $\mathbf{D}$ to be

$$
\mathbf{D} = \begin{bmatrix}
\begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix}_{n\times n} & & \\
& \ddots & \\
& & \begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix}_{n\times n}
\end{bmatrix}_{nk\times nk} \qquad (3.12)
$$

The minimisation performed by Denton relies on the assumption that $w_0^{(h)} = 1$ for computational simplicity. Ideally, we could remove this assumption and define

$$\mathbf{D} = \begin{bmatrix} \begin{bmatrix} 0 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix}_{n\times n} & & \\ & \ddots & \\ & & \begin{bmatrix} 0 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix}_{n\times n} \end{bmatrix}_{nk\times nk} \tag{3.13}$$

The problem with such a definition is that $\mathbf{D}'\mathbf{D}$ is singular. Cholette and Chabb (2004) use an approximation to this based on the covariance matrix of an autoregressive model with parameter equal to 0.999.

The elements of $\mathbf{\Pi}$ in (3.1) can be defined as in (3.6) and (3.8) to measure absolute or proportional changes to movement respectively.

3.3 Incorporating Volatility

It could be considered preferable that the post-adjustment method results in larger changes to series that have greater volatility. Such a practice is endorsed by den Butter and Fase (1991) with the reasoning that estimation of seasonality becomes less accurate as variability in the seasonal and irregular components increases. This commonsense approach ensures that the more confidence there is in a seasonally adjusted estimate, the less likely it is that large post-adjustments are applied. Den Butter and Fase (1991) propose to minimise the objective function given by

$$\sum_t \sum_{h=1}^{k} \frac{1}{\sigma_S^{(h)} \sigma_I^{(h)}} \left(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)} \right)^2$$

This can be written in terms of the general expression (3.1) by setting $\mathbf{\Lambda}$ to be the identity matrix and by defining the elements of $\mathbf{\Pi}$ to be

$$\pi_t^{(h)} = \frac{1}{\sigma_S^{(h)} \sigma_I^{(h)}} \tag{3.14}$$

In the case of an additive seasonal decomposition model, $O_t = T_t + S_t + I_t$, the method defined by (3.14) incorporates the relative levels of the component series. This is because the standard error of the seasonal and irregular components, σ_S and σ_I are independent of the level, T_t, of the series. In the case of a multiplicative decomposition model, $O_t = T_t \times S_t \times I_t$, the standard deviation of the seasonal and irregular components, σ_S and σ_I, are standardised. The multiplicative decomposition can be written in an additive decomposition by an appropriate choice of parameters. For notational convenience the time subscript has been removed. This can be shown as follows

$$
\begin{aligned}
O &= T \times S \times I \\
&= T \times (1 + \frac{S^\star}{T}) \times (1 + \frac{I^\star}{S^\star + T}) \\
&= (T + S^\star) \times (1 + \frac{I^\star}{S^\star + T}) \\
&= T + S^\star + I^\star \\
\text{where} & \\
S &= 1 + \frac{S^\star}{T} \Rightarrow S^\star = T \times (S - 1) \\
I &= 1 + \frac{I^\star}{S^\star + T} \Rightarrow I^\star = T \times S \times (I - 1) \\
\Longrightarrow \sigma_{S^\star} &\approx T\sigma_S \\
\sigma_{I^\star} &\approx TS\sigma_S
\end{aligned}
$$

For this reason, in the case of multiplicative decomposition, the level of the series needs to be included explicitly in the objective function.

3.4 Combining Level and Volatility

The objectives concerning level and volatility described in Section 3.1 and 3.3 can be considered simultaneously using the general objective function (3.1) and setting $\mathbf{\Lambda}$ to be the identity matrix and the elements of the matrix $\mathbf{\Pi}$ to be

$$\pi_t^{(h)} = \frac{1}{\sigma_S^{(h)} \sigma_I^{(h)} SA_t^{(h)}} \tag{3.15}$$

$$\pi_t^{(h)} = \frac{1}{\sigma_S^{(h)} \sigma_I^{(h)} \big(SA_t^{(h)}\big)^2} \tag{3.16}$$

We propose a general framework for this case. Consider

$$\pi_t^{(h)} = \left(\alpha \frac{\sigma_S^{(h)} \sigma_I^{(h)}}{\sum_{h=1}^{k} \sigma_S^{(h)} \sigma_I^{(h)}} + \big(1 - \alpha\big) \frac{{SA_t^{(h)}}^2}{\sum_{h=1}^{k} {SA_t^{(h)}}^2} \right)^{-1} \tag{3.17}$$

for a user defined variable $\alpha \in [0,1]$.

The choice of $\pi_t^{(h)}$ described in (3.17) provides a balance between the methods described by (3.8) and (3.14). That is, a compromise between penalising series with high level and series with high variation in the seasonal and irregular components. Note that setting $\alpha = 0$ is equivalent to using the method described by (3.8) and setting $\alpha = 1$ is equivalent to using the method described by (3.14). The choice of α will depend on the user.

3.5 Combining Movement and Volatility

The volatility of the time series can be incorporated into the Denton principle of minimisation of changes to movement by minimising either

$$\min \left[\sum_t \sum_{h=1}^{k} \frac{1}{\sigma_S^{(h)} \sigma_I^{(h)}} \Big(\big(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)} \big) - \big(SA_{t-1}^{(h)} - w_{t-1}^{(h)} SA_{t-1}^{(h)} \big) \Big)^2 \right]$$

or

$$\min \left[\sum_t \sum_{h=1}^{k} \frac{1}{\sigma_S^{(h)} \sigma_I^{(h)}} \Big(w_t^{(h)} - w_{t-1}^{(h)} \Big)^2 \right]$$

which involve absolute and proportional changes to movement respectively. This minimisation can be achieved by using the general form of (3.1) and defining $\mathbf{\Lambda}$ as in Section 3.2 and the elements of $\mathbf{\Pi}$ as in Section 3.4. The general solution (3.2) can then be used.

3.6 Combining Level and Movement

The Denton principle of minimisation of changes to movements is designed for the case of temporal aggregation. In the absence of temporal contraints, the levels of series corrected using the Denton method can become questionable (Cholette and Chhab, 2004). We propose that a possible solution to this is to include both changes to the level and changes to the movement in the objective function of the optimisation.

The joint objectives of minimising changes to movement and minimising changes to level can be integrated in an objective function that is a combination of (3.5) and (3.10). For absolute change this is given by the objective function

$$\begin{aligned} &\lambda \sum_{t \neq 1} \sum_{h=1}^{k} \Big(\big(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)} \big) - \big(SA_{t-1}^{(h)} - w_{t-1}^{(h)} SA_{t-1}^{(h)} \big) \Big)^2 \\ +&(1-\lambda) \sum_{t \neq 1} \sum_{h=1}^{k} \big(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)} \big)^2 \end{aligned}$$

or for proportional change

$$\lambda \sum_t \sum_{h=1}^{k} (w_t^{(h)} - w_{t-1}^{(h)})^2 + (1-\lambda) \sum_t \sum_{h=1}^{k} (w_t^{(h)} - 1)^2$$

where minimisation of each objective function is subject to

$$\sum_{h=1}^{k} w_t^{(h)} SA_t^{(h)} = SA^{(\text{Total\_direct})}$$

where $\lambda \in [0,1]$ is a user-defined value that controls the relative emphasis placed on level and movement.

This can be expressed in the form of (3.1) by setting $\mathbf{\Lambda}$ to be

$$\mathbf{\Lambda} = \lambda \mathbf{D}'\mathbf{D} + (1-\lambda)\mathbf{I} \tag{3.18}$$

The elements of the matrix $\mathbf{\Pi}$ are

$$\pi_t^{(h)} = 1$$

for absolute change or

$$\pi_t^{(h)} = \frac{1}{\left(SA_t^{(h)}\right)^2}$$

for proportional change.

To obtain a reasonable balance between level and movement, a value of λ close to 1 needs to be chosen as $\left(w_t^{(h)} - w_{t-1}^{(h)}\right)^2$ in general will be a lot smaller than $\left(w_t^{(h)} - 1\right)^2$. When $\lambda \neq 1$, the form of the differencing matrix $\mathbf{D}$ shown in (3.13) can be used as $\mathbf{\Lambda}$ will not be singular. For $\lambda = 1$, the solution requires either the form of $\mathbf{D}$ shown in (3.12) or the method used by Cholette and Chhab (2004) as described in Section 3.2.

3.7 Combining Level, Movement and Volatility

The joint objectives of minimising changes to movement and level and taking account of volatility can be incorporated in one objective function, using appropriate choices of the matrices $\mathbf{\Lambda}$ and $\mathbf{\Pi}$ in (3.1). That is, choosing weights $w_t^{(h)}$ to minimise the objective function

$$\lambda \sum_t \sum_{h=2}^{k} \pi_t^{(h)} \Big(\big(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}\big) - \big(SA_{t-1}^{(h)} - w_{t-1}^{(h)} SA_{t-1}^{(h)}\big) \Big)^2$$
$$+(1-\lambda) \sum_t \sum_{h=1}^{k} \pi_t^{(h)} \big(SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}\big)^2$$

The matrix $\mathbf{\Pi}$ can have any of the elements $\pi_t^{(h)}$ introduced in Sections 3.1-3.4. This matrix defines the relative emphasis placed on volatility and defines whether the movement or level preservation is in absolute of proportional terms. The matrix $\mathbf{\Lambda}$ (see Section 3.6) defines the balance between preservation of level and movement. The solution given in (3.2) can then be used.

Chapter 4

Assessment of post adjustment corrections

This study was designed to assess the impact of different post-adjustment corrections for systems of directly seasonally adjusted series. This was done for various systems of real and simulated time series with a range of time series characteristics.

4.1 Generating Simulated Data

Simulated time series were derived by simulating the trend, seasonal and irregular components separately and then multiplying them together to obtain the original series. They were simulated with characteristics varying in a way comparable to real data. This was achieved by studying Retail Trade data (see Section 4.2 for more details).

The trend component was simulated by taking a random realisation of an ARIMA(1,1,1) model and estimating it using a cubic spline. The parameters used for the auto-regressive and moving average components were both set to 0.2. Other choices for the order and parameter values for the model could be used. The resulting trend components have a realistic appearance when compared to trend estimates obtained from the Australian Retail Trade series (ABS, 2004b).

The seasonal component was simulated by a sinusoid, where the phase varied from series to series. The overall amplitude of the seasonality in the series was varied and also the amplitude of the seasonality varied between the component series. A large December effect was included in the seasonal component. The seasonal component was allowed to vary over the length of the time series.

The irregular component is simulated by Gaussian noise. The overall amplitude of the irregular component and the degree to which the amplitude varies between series can be controlled in the simulations.

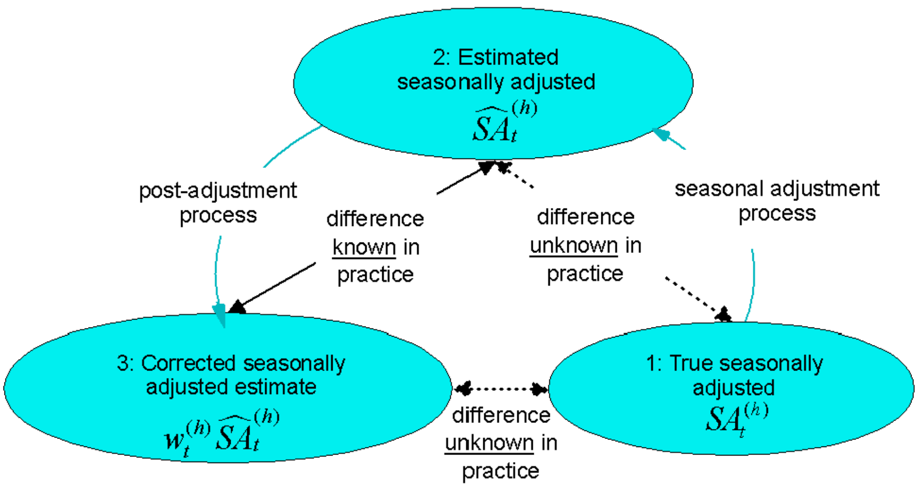

Figure 4.1: Relationship between two seasonally adjusted estimates and the true seasonally adjusted value

The advantages of using simulated data are that the characteristics of the time series in terms of the trend, seasonal and irregular components can be controlled and that the true seasonally adjusted series is known. This knowledge can be helpful in assessing the quality of a post-adjustment correction.

It would be desirable to measure the deviation of the post-adjustment corrected estimate from the true seasonally adjusted value rather than to the estimated seasonally adjusted value estimated from a seasonal adjustment package. This is not possible in practice as the true seasonally adjusted estimate is not known. However, when using simulated series where each of the components of the original (trend, seasonal and irregular) are known. This comparison is possible and informative. Figure 4.1 shows the relationship between the three different types of seasonally adjusted estimates available. The dashed lines represent the comparisons available for a simulation study when the true seasonally adjusted value is known.

Further details of the simulations are available in Appendix A.

4.2 Using Real World Data

The Retail Trade publication (ABS, 2004b) uses data from April 1982 for state and state by industry time series. Australian level data is available from April 1962. Original, seasonally adjusted and trend estimates are published at a state by industry level (where there are 8 states and 18 industries), state level, industry level and Australian totals. The Retail Trade aggregation structure ensures that within each state, seasonally adjusted industry estimates add to the seasonally adjusted state total. The aggregation structure also ensures that the seasonally adjusted state totals add to the seasonally adjusted Australian total. That is, there are two one-dimensional aggregations that have a post-adjustment correction. However, seasonally adjusted industry estimates within each state are not constrained to add to the seasonally adjusted industry totals. It would be preferable to have a two-dimensional post-adjustment method that accounted for states and industries simultaneously. The Retail Trade system of series will be used to illustrate the implementation of the method chosen from the simulation study.

4.3 Results

For each simulated series, multiple methods of post-adjustment correction were applied. Each method used is a special case of the general form of the objective function shown in (3.1). This solution is

$$\widetilde{\mathbf{sa}} = \mathbf{sa} + \mathbf{\Pi}^{-\frac{1}{2}}\mathbf{\Lambda}^{-1}\mathbf{\Pi}^{-\frac{1}{2}}\mathbf{B}(\mathbf{B}'\mathbf{\Pi}^{-\frac{1}{2}}\mathbf{\Lambda}^{-1}\mathbf{\Pi}^{-\frac{1}{2}}\mathbf{B})^{-1}[\mathbf{sa}^{(Total\_direct)} - \mathbf{B}'\,\mathbf{sa}]$$

where different methods were defined by choices of λ and the elements of $\mathbf{\Pi}$, $\pi_t^{(h)}$.

The value of λ defines the relative importance of the changes to estimates of seasonally adjusted level and of seasonally adjusted movement in the calculation of the weights, as shown in (3.18).

The choice of $\mathbf{\Pi}$ defines the relative importance of volatility and whether real or proportional level and movement changes are used.

The choices used were

$$\begin{aligned}
\lambda &\in \{0.00, 0.50, 0.9, 0.99, 0.999, 1.00\} \\
\pi_t^{(h)} &= 1 \\
\pi_t^{(h)} &= \frac{1}{SA_t(h)} \\
\pi_t^{(h)} &= \left(\alpha \frac{\sigma_S^{(h)}\sigma_I^{(h)}}{\sum_{h=1}^k \sigma_S^{(h)}\sigma_I^{(h)}} + (1-\alpha)\frac{{SA_t^{(h)}}^2}{\sum_{h=1}^k {SA_t^{(h)}}^2}\right)^{-1} \\
\text{where} \quad \alpha &\in \{0.0, 0.1, \ldots, 0.9, 1.0\} \\
\pi_t^{(h)} &= \frac{1}{\sigma_S^{(h)}\sigma_I^{(h)} SA_t^{(h)}} \\
\pi_t^{(h)} &= \frac{1}{\sigma_S^{(h)}\sigma_I^{(h)} (SA_t^{(h)})^2}
\end{aligned}$$

The quantities $\sigma_S^{(h)}$ and $\sigma_I^{(h)}$ are known from the simulations but have to be estimated for the purposes of calculating the weights $w_t^{(h)}$. The estimation of $\sigma_I^{(h)}$ is straightforward as the expected value of the irregular component is 1. The estimate used was

$$\hat{\sigma}_I^{(h)} = \frac{1}{n}\sum_{t=1}^{n}(\hat{I}_t^{(h)} - 1)^2$$

The estimation of $\hat{\sigma}_S^{(h)}$ is less obvious. The method chosen was to use the mean annual variation of the estimated seasonal component, that is

$$\hat{\sigma}_S^{(h)} = \frac{1}{p(y-1)}\sum_{i=1}^{p}\sum_{j=2}^{y}\left|\frac{\hat{S}_{i,j}^{(h)} - \hat{S}_{i,j-1}^{(h)}}{\hat{S}_{i,j-1}^{(h)}}\right|$$

where, here, the subscripts of the seasonal factors $\hat{S}_{i,j}^{(h)}$ refer to period ($i = 1, 2, \ldots, p$) and year ($j = 1, 2, \ldots, y$). Both $\sigma_S^{(h)}$ and $\sigma_I^{(h)}$ were assumed to be constant over time for each series. This assumption is valid for the simulated data but would need to be evaluated in practice.

The results presented in Sections 4.3.1-4.3.3 relate to a number of quality measures. These quality measures involve the level, movement and volatility of the series. For each method, the quality measure is given relative to the current method of proration. The quality measures are given when they have been averaged over all systems of simulated series (Section 4.3.1), averaged over those systems with heterogenous simulated component series (Section 4.3.2) and for the ABS Retail Trade data (Section 4.3.3). The full results are in Appendix B.

4.3.1 All Systems of Simulated Component Series

The available results over all systems of simulated series display the following major characteristics

1. Quality measure used were the sums of squares of proportional changes to seasonally adjusted level and movement estimates. These were both weighted and unweighted by the volatility in the seasonal and irregular components. For all of these measures, the option of choosing $\pi_t^{(h)} = \frac{1}{(SA_t^{(h)})^2}$ gives reductions of around 15% when compared to the current method of proration used by the ABS. This is to be expected as some reductions are inevitable considering the current method of proration does not minimise the sums of proportional changes.

2. The reductions described in point 1 are fairly steady for values of λ up to 0.99, but the changes to level can be very large for $\lambda = 1$. Choices of λ up to 0.99 have very little impact in terms of changes to level or movement. It is also clear that choosing $\lambda = 1$ is very undesirable in terms of changes to the levels of the component series. The large

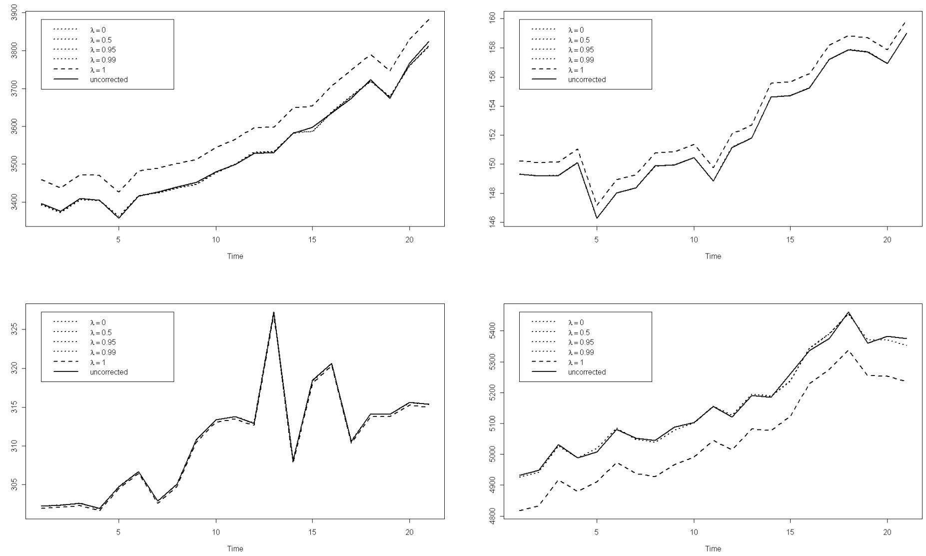

Figure 4.2: An example of the corrected series for different values of λ .

changes to the level of the time series when $\lambda = 1$ is used can be seen clearly in Figure 4.2.

3. Two measures of changes to movement were given in Section 3.2. These were of the form proposed by Denton (1971) in (3.10) and the preferable, but non- linear, form shown in (3.11). When evaluating methods against these measures there was very little difference between the two. This similarity was suggested by Di Fonzo and Marini (2003).

4. When the quality measure of interest includes the volatility of the irregular and seasonal components of the time series, methods of correction that account for this volatility have improvements of about 30% compared to the current method of proration. When the quality measure used in evaluation does not include the volatility, methods of correction that use the volatility perform similarly to the current method of proration.

5. For quality measures that take into account the volatility of the seasonal and irregular components of the time series, there is a local minimum in the quality measure for values of α near 0.2. This can be seen in the bottom right plot of Figure 4.3. These options were inferior to the global minimum of setting $\pi_t^{(h)} = \frac{1}{(SA_t^{(h)})^2}$.

The full tables of results are in Appendix B.1.

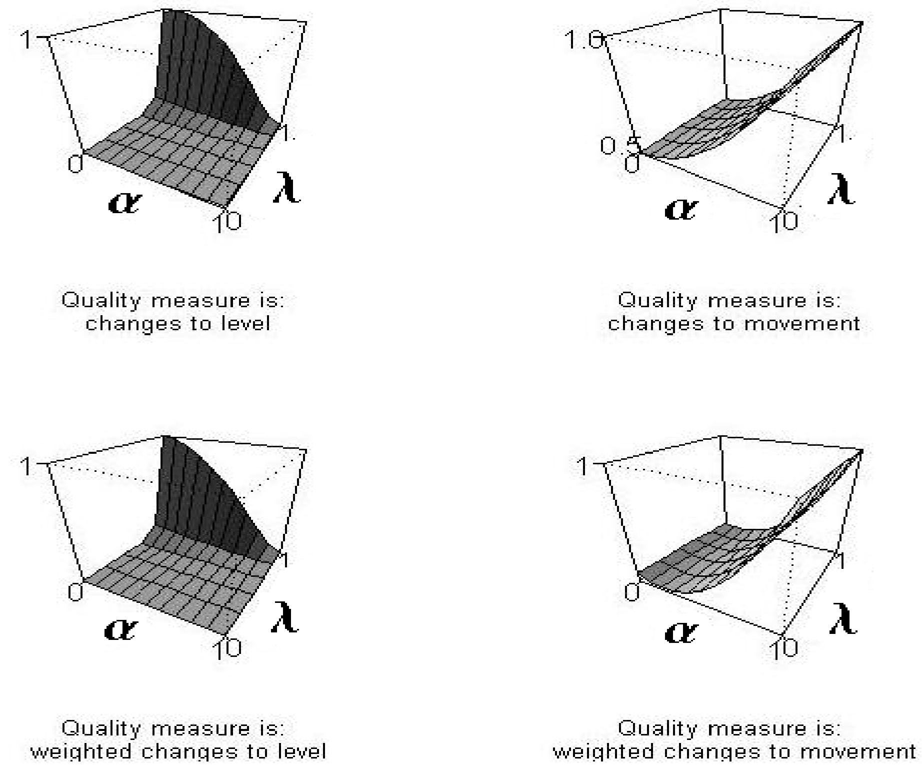

Figure 4.3: The influence of α and λ on four quality measures when using all simulated series.

4.3.2 Systems of Heterogenous Simulated Component Series

In this instance, systems of series were chosen that were defined to have component series of differing characteristics. The results displayed the following characteristics.

1. For quality measures that take account of volatility in the seasonal and irregular components of the time series, the inclusion of the volatility in the weights $w_t^{(h)}$ gave improvements of around 35% compared to the current method of proration.

2. There were local minima when the quality measure takes account of volatility. These local minima occurred for values of α around 0.4. These minima can be seen in the right hand plots in Figure 4.4. Again, these options were inferior to the global minimum of setting $\pi_t^{(h)} = \frac{1}{(SA_t^{(h)})^2}$.

3. There were slight improvements to quality measures involving changes to movement when the value of λ was set to 1. Again, such options led to undesirable outcomes in terms of changes to level.

The full tables of results are in Appendix B.2.

4.3.3 Retail Data

For this study, Australian Retail Trade data was used. An aggregation structure was imposed to ensure that state totals add to Australian level totals. The results displayed the following characteristics.

1. The results show similar patterns to the simulated data. However, the effects of different methods were much more pronounced. Much larger improvements were possible, around 45% for quality measures not weighted by the volatility in the seasonal and irregular components of the time series. The changes to the level of the seasonally adjusted estimates were very large when λ was set to 1.

2. Most interestingly, even when the quality measure takes account of the volatility in the seasonal and irregular components of the time series, there is very little improvement by taking account of this volatility when calculating the weights $w_t^{(h)}$. This points to some fundamental difference between the Retail data and the simulated series. This is to be expected as the simulated systems of time series were designed to cover a large range of characteristics, whereas the Retail Trade series are only one system of series.

The full tables of results are in Appendix B.3.

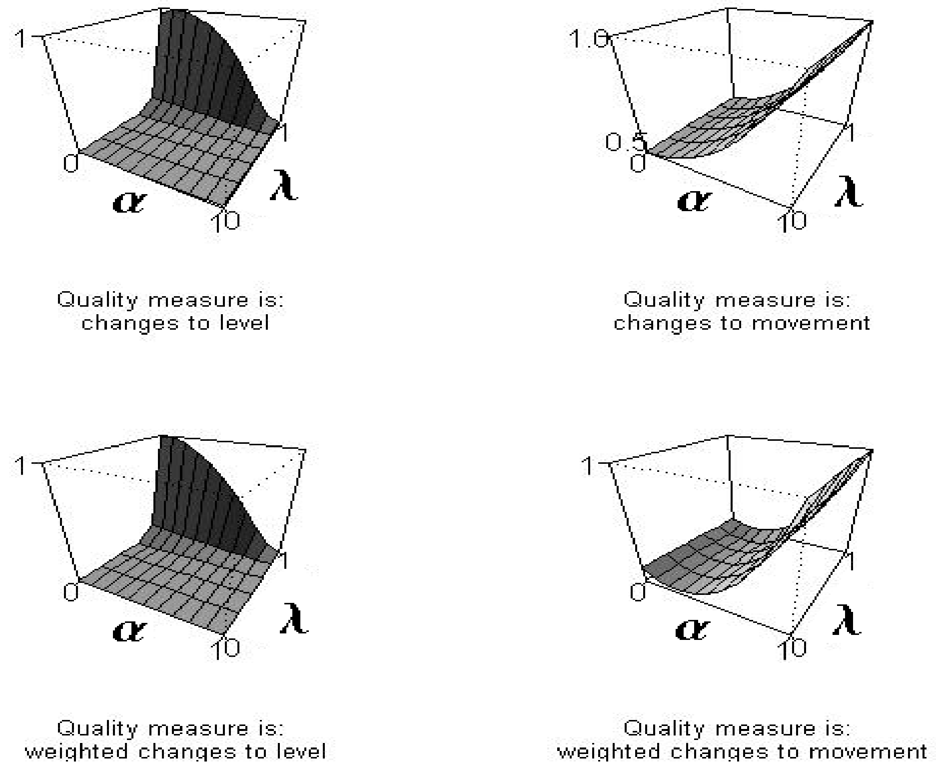

Figure 4.4: The influence of α and λ on four quality measures when using heterogenous simulated series.

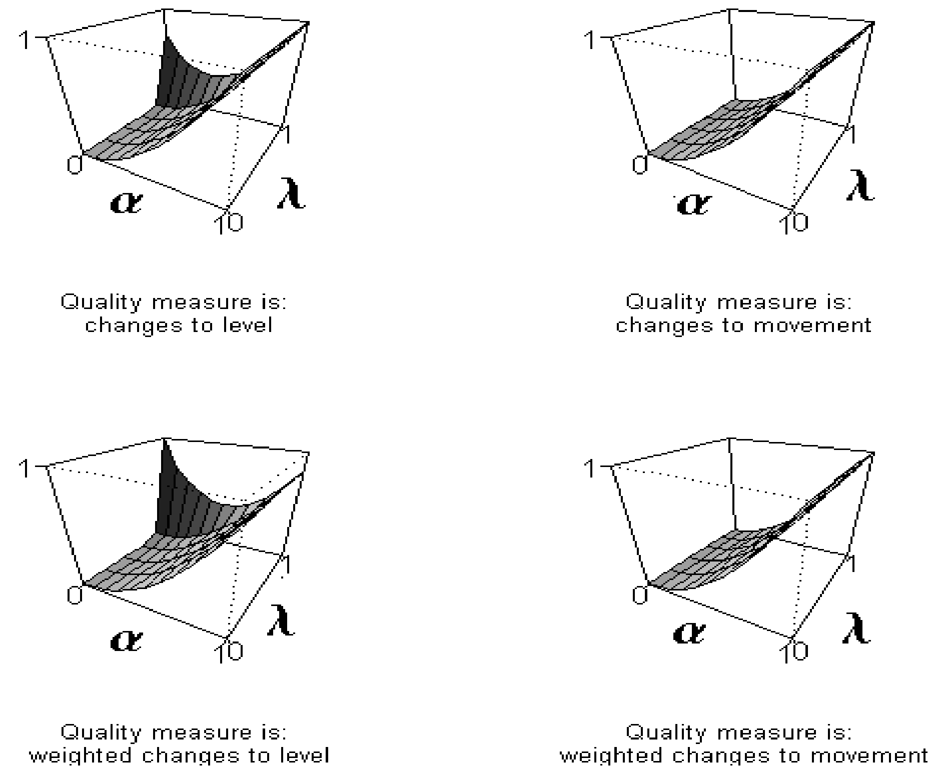

Figure 4.5: The influence of α and λ on four quality measures when using Retail Trade series.

Chapter 5

Conclusions, Recommendations and Questions

5.1 Conclusions

Minimising changes to the level of seasonally adjusted estimates, versus minimising changes to the movement of seasonally adjusted estimates

1. Minimising proportional changes to the level of seasonally adjusted estimates also leads to reductions in the proportional changes to period to period seasonally adjusted movement estimates compared to the current method of proration used by the ABS.
2. Explicitly minimising changes to seasonally adjusted movement estimates gives minor improvements, compared to minimising proportional changes to seasonally adjusted level estimates. This can be seen by the small influence of the choice of λ, as defined in (3.18), on each of the quality measures.
3. Calculating the weights $w_t^{(h)}$ based on seasonally adjusted level estimates alone leads to large reductions in changes to seasonally adjusted movement estimates. This is important because setting $\lambda > 0$ adds the dimension of time into the calculation of the weights $w_t^{(h)}$. Such calculations are computationally intensive.

The role of volatility of the seasonal and irregular components of the time series.

1. There seems to be little value in accounting for $\sigma_S^{(h)}$ and $\sigma_I^{(h)}$ in the calculation of the weights $w_t^{(h)}$.

2. The compromise of setting $\alpha = 0.2$ in (3.17) appears to be a reasonable choice given the simulated data, but this does not give good results for the Australian Retail Trade data.

5.2 Recommendations

Given our initial results, we recommend that the method to be implemented for the ABS production environment be to minimise

$$\sum_t \sum_{h=1}^{k} \Big(\frac{SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}}{SA_t^{(h)}} \Big)^2$$

That is, to set $\alpha = 0$ (see (3.17)) and $\lambda = 0$ (see (3.18)).

This recommendation is based on the quality measures that have been studied in this paper. The general form of the objective function gives scope to improve performance with respect to other quality measures that may be deemed more desirable.

5.3 Questions for the Methodology Advisory Committee

The methods presented in this paper are motivated by a desire to take into account the levels, movements and the volatility of component time series, while still ensuring that systems of time series are additive for a system of seasonally adjusted estimates.

We would appreciate responses to the questions

1. Is additivity in systems of seasonally adjusted estimates desirable from a methodological perspective?

2. What is the relative importance of each of level, movement and volatility?

3. Are the penalty functions sensibly constructed? Are there any others more appropriate?

4. What other avenues should be explored before a method is chosen for application to ABS published data?

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Appendix A

Simulation Details

Series were simulated using the multiplicative decomposition model:

$$O_t = T_t \times S_t \times I_t$$

where O_t, T_t, S_t and I_t refer to the original estimates, trend, seasonal and irregular components respectively at time t.

$\mathbf{T_t}$: Method for Simulating Trend series

The method used was to simulate a series using an ARIMA model and then to smooth this using a cubic spline.

The choice of what order ARIMA model to use is not clear. By having an I(1) component there should be an overall direction to the series. By having fewer simulated points from the ARIMA model than spline points there should be fewer turning points in the resulting series. the followng is the R code used to simulate the trend component.

```
trend<-
spline(arima.sim((yr*period/12),model=list(ar=0.2,ma=0.2,ndiff=1))
,n=2*yr*period
)
tsplot(trend$y[1:(yr*period)])
sim[,,"T",i]<-matrix( (trend$y[1:(yr*period)]-
min(trend$y[1:(yr*period)])+1)*level,ncol=period,byrow=T )
```

Example trend series are shown in Figure A.1.

$\mathbf{S_t}$: Method for Simulating Seasonal series

- **defc** is a parameter to control the degree to which the strength of the seasonal component is allowed to vary between series;

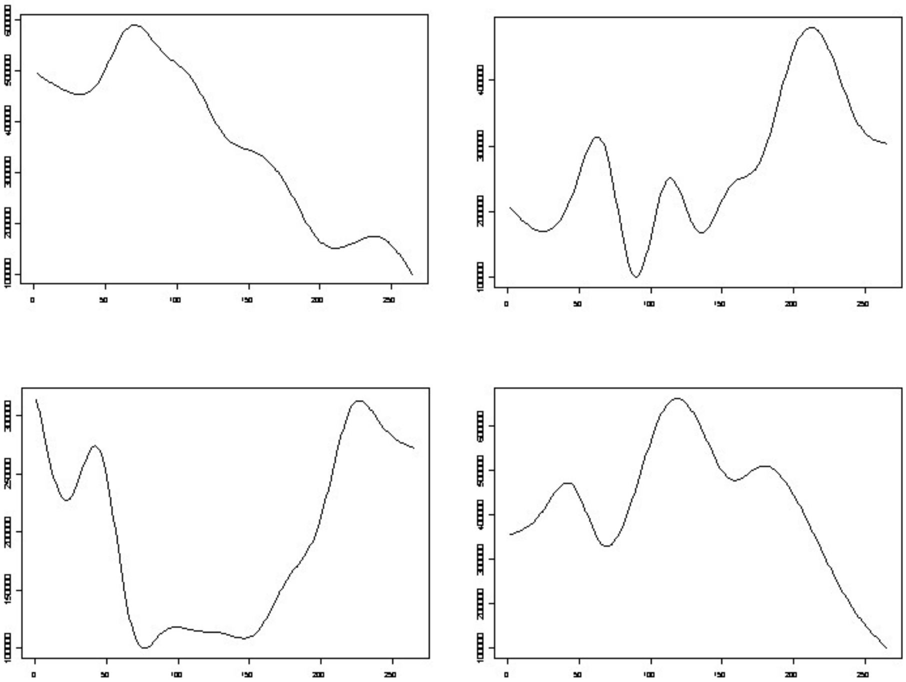

Figure A.1: Examples of trend series produced by simulations.

- **defd** as a parameter controlling the overall strength of the seasonal component;

- **drift** is a parameter controlling the degree to which there is moving seasonality.

The initial seasonal component takes a sine wave centred around one, with an amplitude of **defd** $\pm$ **defc**.

The parameter **defc** also defines the degree to which the seasonal patterns of the component series are synchronised. A low value of **defc** gives seasonal cycles closer to synchronism between the seasonal pattern of component series than a high value.

The **drift** parameter effects the period of the sine wave. A higher value of **drift** leads to a higher degree of moving seasonality in the series.

The seasonal component for December can be made to vary between 1.3 and 1.5 time the average of the other months to represent the effect of Christmas sales in the Retail Trade series.

The seasonal components are then moderated so that the sum over each year adds to twelve (or 4 for quarterly data)

The following is the R code used to simulate the seasonal component.

```
Temp <- matrix( 1.00 +(runif(1,min=(defd*(1-
defc)),max=(defd*(1+defc))))* sin(2*pi*(time-5*runif(1,min=0,
max=defc))*(1-
drift)/period), ncol=period,byrow=T)

temp[,12] <- (runif(1,min=1.3,max=1.5)**december)*
(mean(temp[,1:11])/mean(temp[,12]))* temp[,12]

texttt{sim[,,"S",i] <- temp*
Kronecker(period/rowSums(temp),matrix(1,nrow=1,ncol=period)))
```

In the systems of simulated series, these parameters took on the following values.

$$\text{defc} \in \{0.001, 0.150, 0.300, 0.450, 0.60\}$$

$$\text{defd} \in \{0.05, 0.10, 0.15, 0.20, 0.25\}$$

$$\text{drift} \in \{0.000, 0.005, 0.010, 0.015\}$$

I_t: Method for Simulating Irregular series

defe is a parameter controlling the overall strength of the irregular component and deff is a parameter to control the degree to which the strength of the irregular component is allowed to vary between series.

The irregular component is defined as independent observations from a Normal distribution with mean of one (for multiplicative series) and standard deviation beginning at defe and increasing in increments of deff for each component series.

```
sim[,,"I",i] <-matrix(rnorm((period*yr), mean=1, sd=defe+deff*i),
ncol=period,byrow=T)}
```

In the systems of simulated series, these parameters took on the following values:

$$\text{defe} \in \{0.005, 0.006, 0.007, 0.008, 0.009\}$$

$$\text{deff} \in \{0.000, 0.001, 0.002, 0.003, 0.004\}$$

Appendix B

Tables of Results

B.1 All Systems of Simulated Data

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.64	1.64	1.64	1.64	1.64	1.64
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.10	4.45	44.34
	as in (3.17)	$\alpha = 0.0$	**0.86**	**0.86**	**0.87**	1.16	8.53	60.30
		$\alpha = 0.1$	**0.87**	**0.87**	**0.87**	1.12	7.95	61.83
		$\alpha = 0.2$	**0.90**	**0.90**	**0.90**	1.10	7.14	60.50
		$\alpha = 0.3$	**0.95**	**0.95**	**0.95**	1.11	6.26	57.60
		$\alpha = 0.4$	1.02	1.02	1.02	1.14	5.35	53
		$\alpha = 0.5$	1.11	1.11	1.11	1.20	4.45	46.51
		$\alpha = 0.6$	1.23	1.23	1.23	1.29	3.59	37.98
		$\alpha = 0.7$	1.38	1.38	1.38	1.41	2.84	27.60
		$\alpha = 0.8$	1.55	1.55	1.55	1.56	2.26	16.26
		$\alpha = 0.9$	1.75	1.75	1.75	1.75	1.94	6.29
		$\alpha = 1.0$	1.99	1.99	1.99	1.99	1.99	1.99
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.23	1.23	1.23	1.39	6.26	54.32
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		1.03	1.04	1.05	1.48	10.71	62.07

Quality measure is $\sum_t \sum_{h=1}^k \left(\frac{SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}}{SA_t^{(h)}}\right)^2$

Table B.1: Sum of Squared Proportional Change in Level. Data is all simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.66	1.66	1.66	1.66	1.66	1.66
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.07	3.75	36.44
	as in (3.17)	$\alpha = 0.0$	**0.84**	**0.84**	**0.85**	1.07	7.12	55.15
		$\alpha = 0.1$	**0.82**	**0.82**	**0.83**	1.01	6.54	54.41
		$\alpha = 0.2$	**0.81**	**0.81**	**0.82**	**0.97**	5.77	51.37
		$\alpha = 0.3$	**0.82**	**0.82**	**0.82**	**0.94**	4.99	47.31
		$\alpha = 0.4$	**0.84**	**0.84**	**0.84**	**0.93**	4.2	42.21
		$\alpha = 0.5$	**0.88**	**0.88**	**0.88**	**0.94**	3.44	36.00
		$\alpha = 0.6$	**0.93**	**0.93**	**0.93**	**0.97**	2.73	28.67
		$\alpha = 0.7$	1.00	1.00	1.00	1.02	2.11	20.40
		$\alpha = 0.8$	1.08	1.08	1.08	1.09	1.62	11.82
		$\alpha = 0.9$	1.19	1.19	1.19	1.19	1.33	4.50
		$\alpha = 1.0$	1.32	1.32	1.32	1.32	1.32	1.32
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		**0.81**	**0.81**	**0.81**	**0.89**	3.63	32.75
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.70**	**0.70**	**0.70**	**0.94**	6.53	43.79

Quality measure is $\sum_t \sum_{h=1}^k \frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}} \left(\frac{SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}}{SA_t^{(h)}}\right)^2$

Table B.2: Weighted Sum of Squared Proportional Change in Level. Data is all simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.63	1.63	1.63	1.63	1.63	1.63
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.00	**0.99**	**0.99**
	as in (3.17)	$\alpha = 0.0$	**0.86**	**0.86**	**0.86**	**0.85**	**0.84**	**0.84**
		$\alpha = 0.1$	**0.87**	**0.87**	**0.87**	**0.86**	**0.86**	**0.86**
		$\alpha = 0.2$	**0.90**	**0.90**	**0.90**	**0.89**	**0.88**	**0.90**
		$\alpha = 0.3$	**0.95**	**0.95**	**0.95**	**0.94**	**0.93**	**0.95**
		$\alpha = 0.4$	1.02	1.02	1.02	1.01	1.01	1.02
		$\alpha = 0.5$	1.11	1.11	1.11	1.11	1.10	1.12
		$\alpha = 0.6$	1.23	1.23	1.23	1.22	1.22	1.23
		$\alpha = 0.7$	1.37	1.37	1.37	1.36	1.36	1.37
		$\alpha = 0.8$	1.54	1.54	1.54	1.53	1.53	1.54
		$\alpha = 0.9$	1.74	1.74	1.74	1.74	1.73	1.73
		$\alpha = 1.0$	1.97	1.97	1.97	1.97	1.97	1.97
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.23	1.23	1.23	1.22	1.21	1.22
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		1.04	1.04	1.04	1.03	1.02	1.02

Quality measure is $\sum_t \sum_{h=1}^{k} \left(w_t^{(h)} - w_{t-1}^{(h)}\right)^2$

Table B.3: Sum of Squared Proportional Change in "Denton" Movement. Data is all simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.65	1.65	1.65	1.65	1.65	1.65
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.00	**0.99**	**0.99**
	as in (3.17)	$\alpha = 0.0$	**0.84**	**0.84**	**0.84**	**0.84**	**0.83**	**0.83**
		$\alpha = 0.1$	**0.82**	**0.82**	**0.82**	**0.82**	**0.81**	**0.82**
		$\alpha = 0.2$	**0.82**	**0.81**	**0.81**	**0.81**	**0.80**	**0.81**
		$\alpha = 0.3$	**0.82**	**0.82**	**0.82**	**0.82**	**0.81**	**0.82**
		$\alpha = 0.4$	**0.84**	**0.84**	**0.84**	**0.84**	**0.83**	**0.84**
		$\alpha = 0.5$	**0.88**	**0.88**	**0.88**	**0.87**	**0.87**	**0.88**
		$\alpha = 0.6$	**0.93**	**0.93**	**0.93**	**0.92**	**0.92**	**0.93**
		$\alpha = 0.7$	**0.99**	**0.99**	**0.99**	**0.99**	**0.99**	**0.99**
		$\alpha = 0.8$	1.08	1.08	1.08	1.08	1.07	1.08
		$\alpha = 0.9$	1.18	1.18	1.18	1.18	1.18	1.18
		$\alpha = 1.0$	1.31	1.31	1.31	1.31	1.31	1.31
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		**0.81**	**0.81**	**0.81**	**0.81**	**0.80**	**0.80**
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.70**	**0.70**	**0.70**	**0.69**	**0.69**	**0.69**

Quality measure is $\sum_t \sum_{h=1}^{k} \frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}} \left(w_t^{(h)} - w_{t-1}^{(h)}\right)^2$

Table B.4: Weighted Sum of Squared Proportional Change in "Denton" Movement. Data is all simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.63	1.63	1.63	1.63	1.63	1.63
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.00	**0.99**	**0.99**
	as in (3.17)	$\alpha = 0.0$	**0.86**	**0.86**	**0.86**	**0.85**	**0.84**	**0.85**
		$\alpha = 0.1$	**0.87**	**0.87**	**0.87**	**0.86**	**0.86**	**0.87**
		$\alpha = 0.2$	**0.90**	**0.90**	**0.90**	**0.89**	**0.89**	**0.90**
		$\alpha = 0.3$	**0.95**	**0.95**	**0.95**	**0.94**	**0.94**	**0.96**
		$\alpha = 0.4$	1.02	1.02	1.02	1.01	1.01	1.03
		$\alpha = 0.5$	1.11	1.11	1.11	1.11	1.10	1.13
		$\alpha = 0.6$	1.23	1.23	1.23	1.22	1.22	1.24
		$\alpha = 0.7$	1.37	1.37	1.37	1.37	1.36	1.38
		$\alpha = 0.8$	1.54	1.54	1.54	1.54	1.53	1.55
		$\alpha = 0.9$	1.74	1.74	1.74	1.74	1.73	1.74
		$\alpha = 1.0$	1.97	1.97	1.97	1.97	1.97	1.97
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.23	1.23	1.23	1.22	1.21	1.23
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		1.04	1.04	1.04	1.03	1.02	1.03

Quality measure is $\sum_t \sum_{h=1}^k \left(\frac{SA_t^{(h)}}{SA_{t-1}^{(h)}} \left(\frac{w_t^{(h)}}{w_{t-1}^{(h)}} - 1 \right) \right)^2$

Table B.5: Sum of Squared Proportional Change in "Ideal" Movement. Data is all simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.65	1.65	1.65	1.65	1.65	1.65
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.00	**0.99**	**0.99**
	as in (3.17)	$\alpha = 0.0$	**0.84**	**0.84**	**0.84**	**0.84**	**0.83**	**0.82**
		$\alpha = 0.1$	**0.82**	**0.82**	**0.82**	**0.82**	**0.81**	**0.82**
		$\alpha = 0.2$	**0.82**	**0.81**	**0.81**	**0.81**	**0.80**	**0.81**
		$\alpha = 0.3$	**0.82**	**0.82**	**0.82**	**0.82**	**0.81**	**0.82**
		$\alpha = 0.4$	**0.84**	**0.84**	**0.84**	**0.84**	**0.83**	**0.84**
		$\alpha = 0.5$	**0.88**	**0.88**	**0.88**	**0.87**	**0.87**	**0.88**
		$\alpha = 0.6$	**0.93**	**0.93**	**0.93**	**0.92**	**0.92**	**0.93**
		$\alpha = 0.7$	**0.99**	**0.99**	**0.99**	**0.99**	**0.99**	1.00
		$\alpha = 0.8$	1.08	1.08	1.08	1.08	1.07	1.08
		$\alpha = 0.9$	1.18	1.18	1.18	1.18	1.18	1.18
		$\alpha = 1.0$	1.31	1.31	1.31	1.31	1.31	1.31
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		**0.81**	**0.81**	**0.81**	**0.81**	**0.80**	**0.81**
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.70**	**0.70**	**0.70**	**0.69**	**0.69**	**0.69**

Quality measure is $\sum_t \sum_{h=1}^k \frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}} \left(\frac{SA_t^{(h)}}{SA_{t-1}^{(h)}} \left(\frac{w_t^{(h)}}{w_{t-1}^{(h)}} - 1 \right) \right)^2$

Table B.6: Weighted Sum of Squared Proportional Change in "Ideal" Movement. Data is all simulations.

B.2 Heterogenous Systems of Simulated Data

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.66	1.66	1.66	1.66	1.66	1.66
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.09	4.04	41.37
	as in (3.17)	$\alpha = 0.0$	**0.85**	**0.85**	**0.86**	1.11	7.01	50.82
		$\alpha = 0.1$	**0.86**	**0.86**	**0.87**	1.08	6.68	53.66
		$\alpha = 0.2$	**0.89**	**0.89**	**0.89**	1.07	6.04	53.09
		$\alpha = 0.3$	**0.94**	**0.94**	**0.94**	1.08	5.35	51.07
		$\alpha = 0.4$	1.02	1.02	1.02	1.12	4.62	47.37
		$\alpha = 0.5$	1.12	1.12	1.12	1.19	3.9	41.76
		$\alpha = 0.6$	1.25	1.25	1.25	1.29	3.22	34.14
		$\alpha = 0.7$	1.41	1.41	1.41	1.43	2.63	24.72
		$\alpha = 0.8$	1.61	1.61	1.61	1.61	2.2	14.51
		$\alpha = 0.9$	1.84	1.84	1.84	1.84	2	5.75
		$\alpha = 1.0$	2.12	2.12	2.12	2.12	2.12	2.12
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.29	1.29	1.29	1.41	5.27	45.35
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		1.05	1.05	1.06	1.39	8.09	47.58

Quality measure is $\sum_t \sum_{h=1}^k \left(\frac{SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}}{SA_t^{(h)}}\right)^2$

Table B.7: Sum of Squared Proportional Change in Level. Data is heterogenous simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.64	1.64	1.64	1.64	1.64	1.64
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.06	3.48	35.50
	as in (3.17)	$\alpha = 0.0$	**0.84**	**0.84**	**0.85**	1.04	6.12	48.02
		$\alpha = 0.1$	**0.81**	**0.81**	**0.82**	**0.99**	5.68	48.41
		$\alpha = 0.2$	**0.80**	**0.80**	**0.80**	**0.94**	5.05	46.33
		$\alpha = 0.3$	**0.79**	**0.79**	**0.79**	**0.90**	4.39	43.24
		$\alpha = 0.4$	**0.80**	**0.80**	**0.81**	**0.88**	3.72	39.02
		$\alpha = 0.5$	**0.83**	**0.83**	**0.83**	**0.89**	3.07	33.56
		$\alpha = 0.6$	**0.88**	**0.88**	**0.87**	**0.91**	2.45	26.84
		$\alpha = 0.7$	**0.94**	**0.94**	**0.94**	**0.95**	1.91	19.07
		$\alpha = 0.8$	1.02	1.02	1.02	1.03	1.49	10.97
		$\alpha = 0.9$	1.13	1.13	1.13	1.13	1.25	4.14
		$\alpha = 1.0$	1.26	1.26	1.26	1.26	1.26	1.26
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		**0.77**	**0.77**	**0.77**	**0.84**	3.17	29.44
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.65**	**0.65**	**0.66**	**0.86**	5.29	35.95

Quality measure is $\sum_t \sum_{h=1}^k \frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}} \left(\frac{SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}}{SA_t^{(h)}}\right)^2$

Table B.8: Weighted Sum of Squared Proportional Change in Level. Data is heterogenous simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
	1		1.62	1.62	1.62	1.62	1.62	1.62
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	**0.99**	**0.98**	**0.99**
		$\alpha = 0.0$	**0.86**	**0.86**	**0.86**	**0.85**	**0.84**	**0.84**
		$\alpha = 0.1$	**0.87**	**0.87**	**0.87**	**0.86**	**0.85**	**0.86**
		$\alpha = 0.2$	**0.90**	**0.90**	**0.89**	**0.89**	**0.88**	**0.90**
		$\alpha = 0.3$	**0.94**	**0.94**	**0.94**	**0.94**	**0.93**	**0.95**
		$\alpha = 0.4$	1.02	1.02	1.01	1.01	1.00	1.03
$\pi_t^{(h)}$	as in (3.17)	$\alpha = 0.5$	1.11	1.11	1.11	1.10	1.09	1.12
		$\alpha = 0.6$	1.23	1.23	1.23	1.23	1.22	1.24
		$\alpha = 0.7$	1.39	1.39	1.38	1.38	1.37	1.39
		$\alpha = 0.8$	1.57	1.57	1.57	1.57	1.56	1.57
		$\alpha = 0.9$	1.79	1.79	1.79	1.79	1.78	1.79
		$\alpha = 1.0$	2.05	2.05	2.05	2.05	2.05	2.05
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.28	1.28	1.28	1.27	1.26	1.27
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		1.06	1.06	1.06	1.05	1.04	1.04

Quality measure is $\sum_t \sum_{h=1}^k \left(w_t^{(h)} - w_{t-1}^{(h)}\right)^2$

Table B.9: Sum of Squared Proportional Change in "Denton" Movement. Data is heterogenous simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
	1		1.62	1.62	1.62	1.62	1.62	1.62
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	**0.99**	**0.99**	**0.99**
		$\alpha = 0.0$	**0.85**	**0.84**	**0.84**	**0.84**	**0.83**	**0.82**
		$\alpha = 0.1$	**0.82**	**0.82**	**0.81**	**0.81**	**0.80**	**0.81**
		$\alpha = 0.2$	**0.80**	**0.80**	**0.80**	**0.79**	**0.79**	**0.80**
		$\alpha = 0.3$	**0.80**	**0.80**	**0.79**	**0.79**	**0.78**	**0.80**
		$\alpha = 0.4$	**0.81**	**0.80**	**0.80**	**0.80**	**0.79**	**0.81**
$\pi_t^{(h)}$	as in (3.17)	$\alpha = 0.5$	**0.83**	**0.83**	**0.83**	**0.83**	**0.82**	**0.84**
		$\alpha = 0.6$	**0.87**	**0.87**	**0.87**	**0.87**	**0.86**	**0.87**
		$\alpha = 0.7$	**0.93**	**0.93**	**0.93**	**0.92**	**0.92**	**0.93**
		$\alpha = 0.8$	1.00	1.00	1.00	1.00	1.00	1.00
		$\alpha = 0.9$	1.10	1.1	1.10	1.1	1.10	1.10
		$\alpha = 1.0$	1.22	1.22	1.22	1.22	1.22	1.22
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		**0.76**	**0.76**	**0.76**	**0.76**	**0.75**	**0.76**
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.66**	**0.66**	**0.65**	**0.65**	**0.64**	**0.64**

Quality measure is $\sum_t \sum_{h=1}^k \frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}} \left(w_t^{(h)} - w_{t-1}^{(h)}\right)^2$

Table B.10: Weighted Sum of Squared Proportional Change in "Denton" Movement. Data is heterogenous simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.62	1.62	1.62	1.62	1.62	1.62
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	**0.99**	**0.98**	**0.99**
	as in (3.17)	$\alpha = 0.0$	**0.86**	**0.86**	**0.85**	**0.85**	**0.84**	**0.84**
		$\alpha = 0.1$	**0.87**	**0.87**	**0.86**	**0.86**	**0.85**	**0.87**
		$\alpha = 0.2$	**0.89**	**0.89**	**0.89**	**0.89**	**0.88**	**0.90**
		$\alpha = 0.3$	**0.94**	**0.94**	**0.94**	**0.94**	**0.93**	**0.96**
		$\alpha = 0.4$	1.01	1.01	1.01	1.01	1.00	1.04
		$\alpha = 0.5$	1.11	1.11	1.11	1.10	1.1	1.14
		$\alpha = 0.6$	1.23	1.23	1.23	1.23	1.22	1.26
		$\alpha = 0.7$	1.38	1.39	1.38	1.38	1.37	1.41
		$\alpha = 0.8$	1.57	1.57	1.57	1.57	1.56	1.58
		$\alpha = 0.9$	1.79	1.79	1.79	1.79	1.79	1.79
		$\alpha = 1.0$	2.05	2.05	2.05	2.05	2.05	2.05
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.28	1.28	1.28	1.27	1.26	1.29
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		1.06	1.06	1.06	1.05	1.04	1.05

Quality measure is $\sum_t \sum_{h=1}^k \left(\frac{SA_t^{(h)}}{SA_{t-1}^{(h)}} \left(\frac{w_t^{(h)}}{w_{t-1}^{(h)}} - 1 \right) \right)^2$

Table B.11: Sum of Squared Proportional Change in "Ideal" Movement. Data is heterogenous simulations.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		1.62	1.62	1.62	1.62	1.62	1.62
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	**0.99**	**0.98**	**0.99**
	as in (3.17)	$\alpha = 0.0$	**0.84**	**0.84**	**0.84**	**0.84**	**0.82**	**0.82**
		$\alpha = 0.1$	**0.82**	**0.81**	**0.81**	**0.81**	**0.80**	**0.81**
		$\alpha = 0.2$	**0.80**	**0.80**	**0.80**	**0.79**	**0.79**	**0.80**
		$\alpha = 0.3$	**0.79**	**0.79**	**0.79**	**0.79**	**0.78**	**0.80**
		$\alpha = 0.4$	**0.80**	**0.80**	**0.80**	**0.80**	**0.79**	**0.81**
		$\alpha = 0.5$	**0.83**	**0.83**	**0.83**	**0.82**	**0.82**	**0.84**
		$\alpha = 0.6$	**0.87**	**0.87**	**0.87**	**0.87**	**0.86**	**0.88**
		$\alpha = 0.7$	**0.93**	**0.93**	**0.93**	**0.92**	**0.92**	**0.94**
		$\alpha = 0.8$	1.00	1.00	1.00	1.00	1.00	1.01
		$\alpha = 0.9$	1.10	1.1	1.10	1.1	1.10	1.10
		$\alpha = 1.0$	1.22	1.22	1.22	1.22	1.22	1.22
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		**0.76**	**0.76**	**0.76**	**0.76**	**0.75**	**0.76**
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.66**	**0.65**	**0.65**	**0.65**	**0.64**	**0.65**

Quality measure is $\sum_t \sum_{h=1}^k \frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}} \left(\frac{SA_t^{(h)}}{SA_{t-1}^{(h)}} \left(\frac{w_t^{(h)}}{w_{t-1}^{(h)}} - 1 \right) \right)^2$

Table B.12: Weighted Sum of Squared Proportional Change in "Ideal" Movement. Data is heterogenous simulations.

B.3 ABS Retail Trade Data

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		26.95	26.95	26.95	26.95	26.95	26.95
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	**0.98**	2.34	51.35
	as in (3.17)	$\alpha = 0.0$	**0.54**	**0.54**	**0.54**	**0.57**	2.01	29.91
		$\alpha = 0.1$	**0.92**	**0.92**	**0.92**	**0.93**	2.05	22.7
		$\alpha = 0.2$	2.06	2.06	2.06	2.06	2.87	19.32
		$\alpha = 0.3$	3.96	3.96	3.96	3.95	4.49	17.10
		$\alpha = 0.4$	6.62	6.62	6.61	6.60	6.92	16.12
		$\alpha = 0.5$	10.04	10.04	10.03	10.02	10.16	16.43
		$\alpha = 0.6$	14.22	14.22	14.21	14.19	14.21	18.07
		$\alpha = 0.7$	19.16	19.15	19.15	19.14	19.07	21.07
		$\alpha = 0.8$	24.85	24.85	24.85	24.84	24.75	25.46
		$\alpha = 0.9$	31.31	31.31	31.31	31.3	31.24	31.28
		$\alpha = 1.0$	38.53	38.53	38.53	38.53	38.53	38.53
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)} SA_t^{(h)}}$		1.95	1.95	1.95	1.92	3.80	79.9
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)} \left(SA_t^{(h)}\right)^2}$		**0.57**	**0.57**	**0.57**	**0.59**	2.52	45.76

Quality measure is $\sum_t \sum_{h=1}^{k} \left(\frac{SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}}{SA_t^{(h)}}\right)^2$

Table B.13: Sum of Squared Proportional Change in Level.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		21.73	21.73	21.73	21.73	21.73	21.73
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	**0.99**	2.34	45.84
	as in (3.17)	$\alpha = 0.0$	**0.74**	**0.74**	**0.74**	**0.79**	2.55	33.76
		$\alpha = 0.1$	**0.96**	**0.96**	**0.96**	**0.99**	2.39	25.22
		$\alpha = 0.2$	1.72	1.72	1.72	1.74	2.79	21.06
		$\alpha = 0.3$	3.04	3.04	3.04	3.04	3.79	17.86
		$\alpha = 0.4$	4.89	4.89	4.89	4.89	5.38	15.72
		$\alpha = 0.5$	7.30	7.30	7.30	7.29	7.57	14.68
		$\alpha = 0.6$	10.25	10.25	10.24	10.23	10.35	14.8
		$\alpha = 0.7$	13.74	13.74	13.74	13.73	13.74	16.11
		$\alpha = 0.8$	17.78	17.78	17.78	17.77	17.73	18.64
		$\alpha = 0.9$	22.37	22.37	22.37	22.36	22.32	22.42
		$\alpha = 1.0$	27.50	27.5	27.50	27.5	27.50	27.50
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)} SA_t^{(h)}}$		1.53	1.53	1.52	1.50	3.20	67.26
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)} \left(SA_t^{(h)}\right)^2}$		**0.71**	**0.71**	**0.71**	**0.75**	2.99	47.45

Quality measure is $\sum_t \sum_{h=1}^{k} \frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}} \left(\frac{SA_t^{(h)} - w_t^{(h)} SA_t^{(h)}}{SA_t^{(h)}}\right)^2$

Table B.14: Weighted Sum of Squared Proportional Change in Level. Data is ABS Retail Trade data.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		26.99	26.99	26.99	26.99	26.99	26.99
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.00	1.00	**0.99**
	as in (3.17)	$\alpha = 0.0$	**0.54**	**0.54**	**0.54**	**0.54**	**0.54**	**0.53**
		$\alpha = 0.1$	**0.92**	**0.92**	**0.92**	**0.92**	**0.91**	**0.91**
		$\alpha = 0.2$	2.06	2.06	2.06	2.06	2.05	2.04
		$\alpha = 0.3$	3.96	3.96	3.96	3.96	3.95	3.93
		$\alpha = 0.4$	6.62	6.62	6.62	6.62	6.61	6.59
		$\alpha = 0.5$	10.05	10.05	10.05	10.04	10.03	10.00
		$\alpha = 0.6$	14.23	14.23	14.23	14.23	14.22	14.19
		$\alpha = 0.7$	19.18	19.18	19.18	19.17	19.16	19.14
		$\alpha = 0.8$	24.88	24.88	24.88	24.88	24.87	24.85
		$\alpha = 0.9$	31.35	31.35	31.35	31.35	31.34	31.34
		$\alpha = 1.0$	38.57	38.57	38.57	38.57	38.57	38.57
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.95	1.95	1.95	1.95	1.95	1.94
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.57**	**0.57**	**0.57**	**0.57**	**0.57**	**0.56**

Quality measure is $\sum_t \sum_{h=1}^k \left(w_t^{(h)} - w_{t-1}^{(h)}\right)^2$

Table B.15: Sum of Squared Proportional Change in "Denton" Movement. Data is ABS Retail Trade data.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		21.77	21.77	21.77	21.77	21.77	21.77
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.00	1.00	**0.99**
	as in (3.17)	$\alpha = 0.0$	**0.74**	**0.74**	**0.74**	**0.74**	**0.73**	**0.73**
		$\alpha = 0.1$	**0.96**	**0.96**	**0.96**	**0.96**	**0.95**	**0.95**
		$\alpha = 0.2$	1.73	1.73	1.73	1.72	1.72	1.71
		$\alpha = 0.3$	3.04	3.04	3.04	3.04	3.03	3.02
		$\alpha = 0.4$	4.90	4.90	4.90	4.90	4.89	4.87
		$\alpha = 0.5$	7.31	7.31	7.31	7.30	7.30	7.28
		$\alpha = 0.6$	10.26	10.26	10.26	10.26	10.25	10.23
		$\alpha = 0.7$	13.76	13.76	13.76	13.76	13.75	13.73
		$\alpha = 0.8$	17.81	17.81	17.81	17.81	17.8	17.79
		$\alpha = 0.9$	22.4	22.4	22.4	22.4	22.4	22.39
		$\alpha = 1.0$	27.54	27.54	27.54	27.54	27.54	27.54
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.53	1.53	1.53	1.52	1.52	1.51
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.71**	**0.71**	**0.71**	**0.71**	**0.70**	**0.70**

Quality measure is $\sum_t \sum_{h=1}^k \frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}} \left(w_t^{(h)} - w_{t-1}^{(h)}\right)^2$

Table B.16: Weighted Sum of Squared Proportional Change in "Denton" Movement. Data is ABS Retail Trade data.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		27.12	27.12	27.12	27.12	27.12	27.12
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.00	1.00	**0.98**
	as in (3.17)	$\alpha = 0.0$	**0.54**	**0.54**	**0.54**	**0.54**	**0.54**	**0.54**
		$\alpha = 0.1$	**0.92**	**0.92**	**0.92**	**0.92**	**0.92**	**0.91**
		$\alpha = 0.2$	2.06	2.06	2.06	2.06	2.05	2.04
		$\alpha = 0.3$	3.96	3.96	3.96	3.96	3.95	3.93
		$\alpha = 0.4$	6.63	6.63	6.63	6.62	6.61	6.58
		$\alpha = 0.5$	10.06	10.06	10.06	10.06	10.05	10.01
		$\alpha = 0.6$	14.27	14.27	14.27	14.27	14.25	14.22
		$\alpha = 0.7$	19.25	19.25	19.25	19.24	19.23	19.20
		$\alpha = 0.8$	25.00	25.00	25.00	25.00	24.99	24.97
		$\alpha = 0.9$	31.54	31.54	31.54	31.53	31.53	31.52
		$\alpha = 1.0$	38.85	38.85	38.85	38.85	38.85	38.85
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.95	1.95	1.95	1.95	1.95	1.93
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.57**	**0.57**	**0.57**	**0.57**	**0.57**	**0.56**

Quality measure is $\sum_t \sum_{h=1}^k \left(\frac{SA_t^{(h)}}{SA_{t-1}^{(h)}} \left(\frac{w_t^{(h)}}{w_{t-1}^{(h)}} - 1 \right) \right)^2$

Table B.17: Sum of Squared Proportional Change in "Ideal" Movement. Data is ABS Retail Trade data.

			λ					
			0	0.5	0.9	0.99	0.999	1
$\pi_t^{(h)}$	1		21.87	21.87	21.87	21.87	21.87	21.87
	$\frac{1}{SA_t^{(h)}}$		1.00	1.00	1.00	1.00	1.00	**0.99**
	as in (3.17)	$\alpha = 0.0$	**0.74**	**0.74**	**0.74**	**0.74**	**0.74**	**0.74**
		$\alpha = 0.1$	**0.96**	**0.96**	**0.96**	**0.96**	**0.96**	**0.95**
		$\alpha = 0.2$	1.72	1.72	1.72	1.72	1.72	1.71
		$\alpha = 0.3$	3.04	3.04	3.04	3.03	3.03	3.01
		$\alpha = 0.4$	4.90	4.90	4.90	4.90	4.89	4.86
		$\alpha = 0.5$	7.31	7.31	7.31	7.31	7.30	7.27
		$\alpha = 0.6$	10.28	10.28	10.27	10.27	10.26	10.23
		$\alpha = 0.7$	13.79	13.79	13.79	13.79	13.78	13.76
		$\alpha = 0.8$	17.87	17.87	17.87	17.87	17.86	17.84
		$\alpha = 0.9$	22.51	22.51	22.51	22.51	22.5	22.49
		$\alpha = 1.0$	27.7	27.7	27.7	27.7	27.7	27.7
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}SA_t^{(h)}}$		1.52	1.52	1.52	1.52	1.52	1.50
	$\frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}\left(SA_t^{(h)}\right)^2}$		**0.71**	**0.71**	**0.71**	**0.71**	**0.71**	**0.71**

Quality measure is $\sum_t \sum_{h=1}^k \frac{1}{\sigma_I^{(h)}\sigma_S^{(h)}} \left(\frac{SA_t^{(h)}}{SA_{t-1}^{(h)}} \left(\frac{w_t^{(h)}}{w_{t-1}^{(h)}} - 1 \right) \right)^2$

Table B.18: Weighted Sum of Squared Proportional Change in "Ideal" Movement. Data is ABS Retail Trade data.