Question 1.

Draw all the non-isomorphic simple connected graphs on 4 vertices. [Think first about what systematic approach you are going to use to make sure that you get all of the required graphs without repetitions. You could draw all the non-isomorphic graphs and then search amongst them for the connected ones, but there is probably a quicker way.]

Question 2.

Label the vertices and edges of the following graphs in any convenient way and show whether or not they are isomorphic.

![Graph 1](image1.png)

![Graph 2](image2.png)

**Question 3.**

The floor plan of a one-storey building is shown below. There are 4 rooms, A, B, C, D, and doorways are indicated between rooms, and to the outside O. By converting the question to a problem in graph theory, decide if it is possible to find a path that starts in room A and passes through every doorway exactly once, ending in room C. Write down such a path if one exists.
Question 4.

Apply Kruskal’s algorithm to the graph below to find a spanning tree for the graph that has minimum weight.

![Graph Image]

Question 5.

(a) Use the quotient-remainder theorem to show that any integer $n$ can be written in one of the three forms $n = 3q$ or $n = 3q + 1$ or $n = 3q + 2$.

(b) By considering the three cases from (a), prove that the square of any integer has the form $3k$ or $3k + 1$ for some integer $k$.

Question 6.

Use proof by contradiction to prove that for any integer $n$, $n^2 - 2$ is not divisible by 4. [Hint: Consider the two cases where $n$ is even and $n$ is odd.]
Question 7.

(a) By writing 174 and 835 as products of prime factors, find lcm(174, 835).

(b) Find the greatest common divisor of 3138 and 176. Hence, or otherwise, find integers m and n such that

\[ \gcd (3138, 176) = 3138m + 176n. \]

Question 8.

Use the Sieve of Eratosthenes to find all of the prime numbers between 300 and 400. [Working must be shown as per Section 4.5 of your notes.]