

Techniques for Mental Arithmetic

Addition

Grouping

When we add up a list of numbers, it is OK to change the order of the numbers and to perform the addition operations in any order. For instance $1+2+3+4 = 1+3+4+2 = (1+3) + (4+2) = 1 + (3+4+2) = 10$. Sometimes we can use this to group numbers to make our additions easier.

$$3 + 4 + 5 + 6 + 7 = (3 + 7) + (4 + 6) + 5 = 10 + 10 + 5 = 25$$

Here we have found two pairs which add up to 10, $3 + 7$ and $4 + 6$. (Work out all the pairs which do this!) By doing these additions first we make the whole calculation easier.

Question 1: Try regrouping the following additions using 10-pairs (pairs of numbers which add to 10) and then adding them.

a) $5 + 6 + 5 + 4 =$ b) $3 + 8 + 1 + 7 + 9 + 2 =$ c) $6 + 6 + 3 + 4 + 7 =$

d) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 =$ e) $8 + 7 + 8 + 3 + 2 + 6 + 4 =$

You can also group larger numbers to make your mental calculations easier. For instance $23 + 45 + 17 = (23 + 17) + 45 = 40 + 45 = 85$

Question 2: Try the following mental calculations:

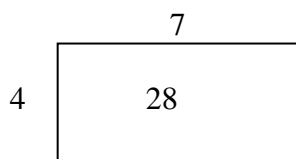
a) $16 + 14 + 13 =$ b) $12 + 19 + 8 =$ c) $31 + 46 + 19 =$

d) $37 + 6 + 13 + 14 =$ e) $21 + 22 + 28 + 19 =$

Multiplication using Areas

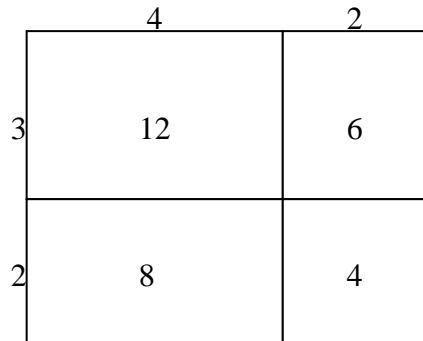
Using this technique, you should be able to learn to multiply any pair of two-digit numbers together in your head. (If you know your tables up to 9×9)

First we need to explore the connection between multiplication and calculating areas. The area of a rectangle can be found by multiplying its length by its width:



This area has length 7, width 4 and area $7 \times 4 = 28$

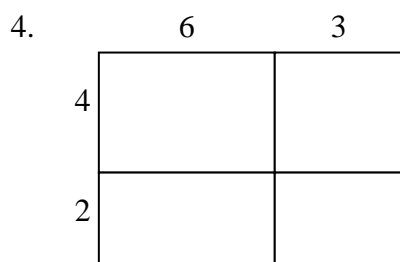
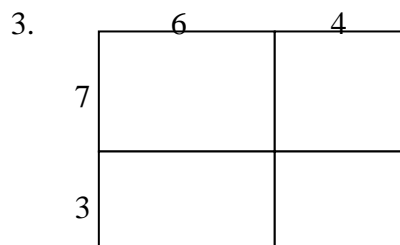
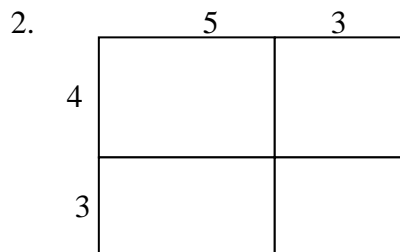
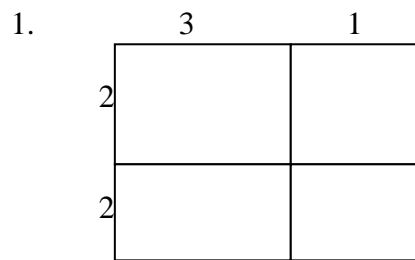
The good thing is that if a rectangle is broken up into smaller rectangles, it doesn't matter whether the areas of the smaller rectangles are added up to form the total area or whether we add the sides and then multiply to get total area. Look at this example:



The area of this shape can be found by calculating the separate areas of the small rectangles and then adding them:
 $12 + 6 + 8 + 4 = 30$

Or by first finding the total length:
 $4 + 2 = 6$ and the total width:
 $3 + 2 = 5$ and then multiplying to find the total area: $6 \times 5 = 30$

Use both of these methods to find the areas of the following rectangles (not drawn to scale). Show your working.



Multiplication of numbers ending with zeros

If you know your tables, you should be able to do simple calculations involving numbers which end with a zero. For instance, if you know that $3 \times 7 = 21$ then you should be able to see why $30 \times 7 = 210$ and $3 \times 70 = 210$. If both of the numbers you are multiplying together end in a 0, then there will be two zeros on the end of the answer. For instance $1 \times 1 = 1$ and $10 \times 10 = 100$. Similarly $30 \times 70 = 2100$

See if you can write down the answers to the following calculations using your tables and the above rules for dealing with end zeros.

1. 2×20
2. 4×50
3. 70×6
4. 20×9
5. 8×70
6. 9×90
7. 30×30
8. 50×50
9. 40×70
10. 60×30
11. 80×80
12. 90×4

Multiplying a two-digit number by a one-digit number

Using a combination of our areas method and the rules above for multiplying numbers ending in a zero, we can simplify a multiplication of a two-digit number by a one-digit number as follows:

$$7 \times 23$$

Break 23 up as $20 + 3$

	20	3
7	140	21

$7 \times 20 = 140$
 $3 \times 7 = 21$

Adding together results we get $140 + 20 + 1 = 161$, so $7 \times 23 = 161$

Try doing the following multiplications using this method:

1. 3×45

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2. 6×64

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3. 8×17

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4. 2×39

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5. 7×92

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6. 4×88

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7. 9×46

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8. 5×78

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Multiplying a two-digit number by another two-digit number

We use a combination of techniques we have already learnt as follows:

$$23 \times 42 = (20 + 3) \times (40 + 2)$$

	20	3	
40	800	120	
2	40	6	

$$800 + 120 + 40 + 6 = 966$$

Try the following:

1. 12×17

2. 44×52
