



Developing representational competence



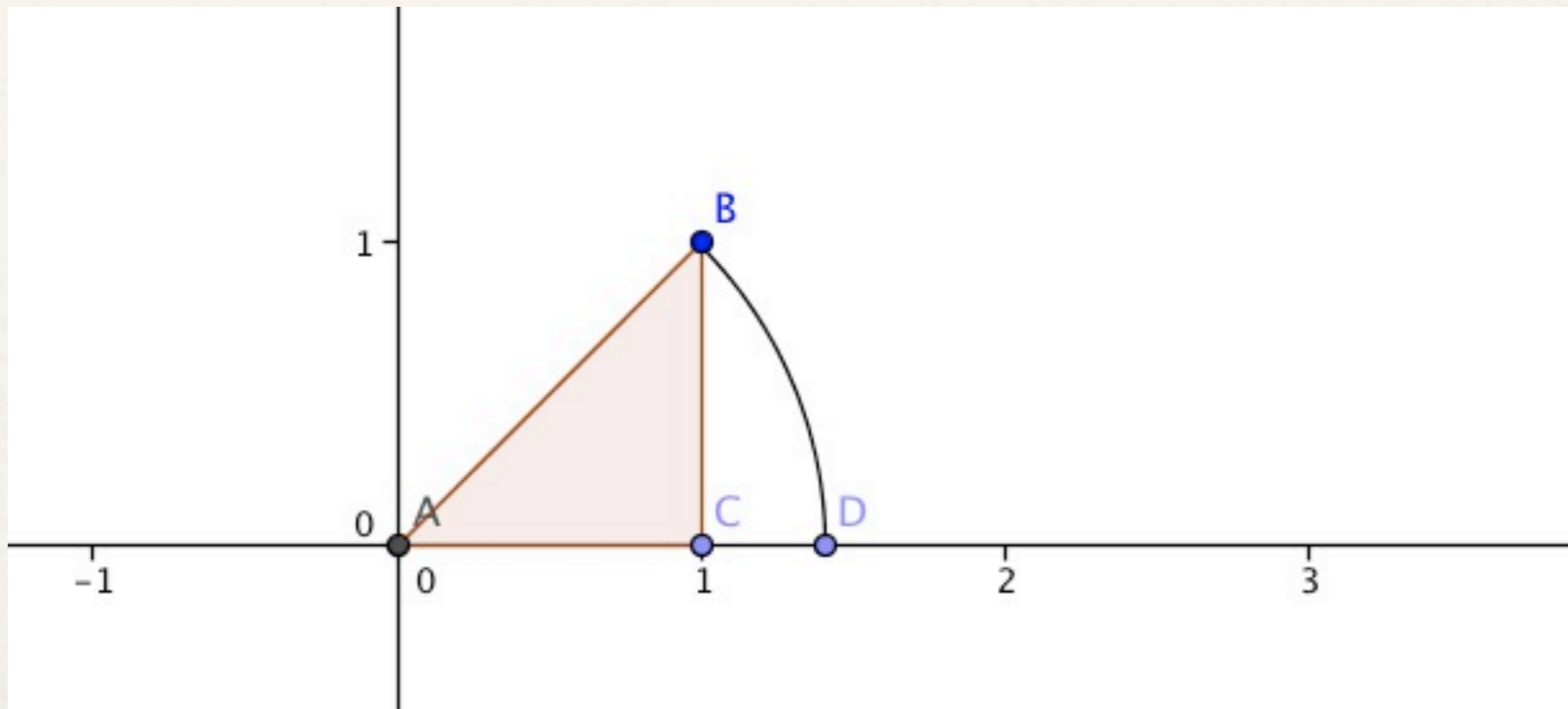
Peter Gould 2009

What are representations?

We represent numbers by points on a line or by rows of counters.

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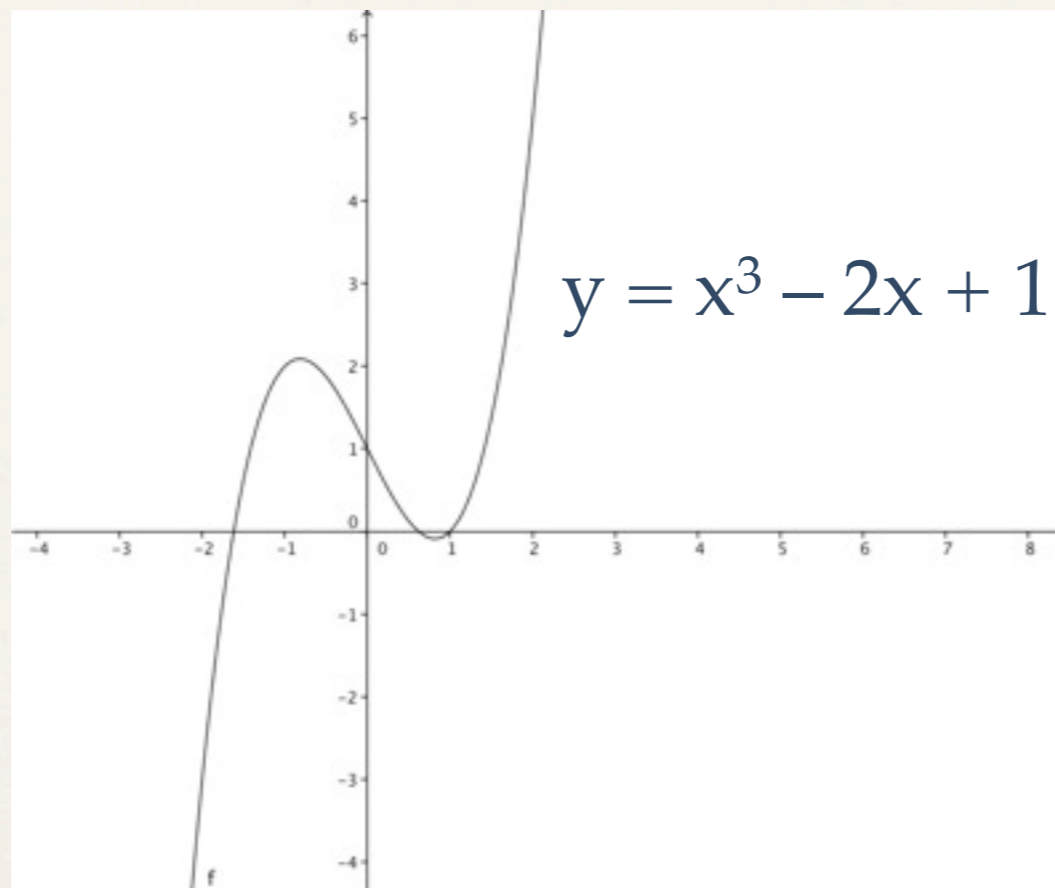


What are representations?

We use equations and curves on a plane to represent each other.

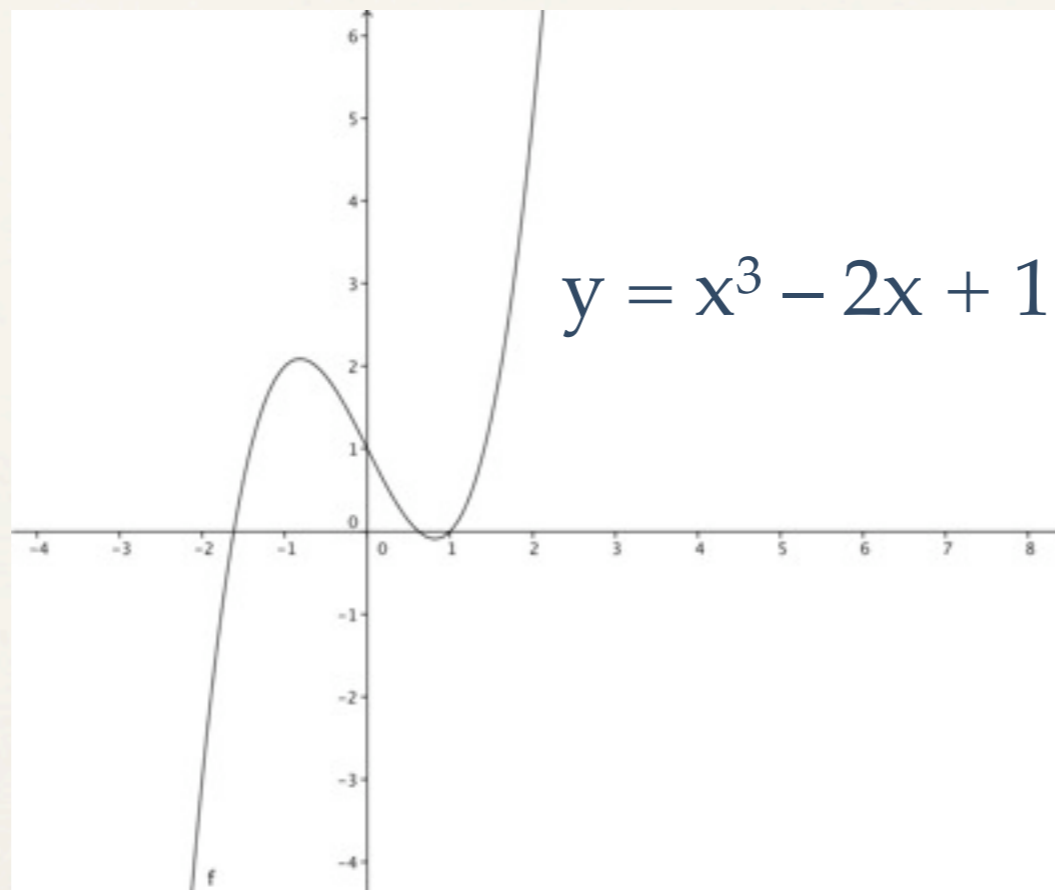
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What are representations?

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So what do we mean by a representation?

What are representations?

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- ▶ A representation is not the thing being represented nor is it the materials used in forming a representation.

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- ▶ A representation is not the thing being represented nor is it the materials used in forming a representation.
 - Multi-base Arithmetic Blocks (MAB) are not the base-10 system.

Presenting and representing

Presenting and representing

$$(x + 3)(x + 2)$$

Presenting and representing

$$(x + 3)(x + 2)$$

x

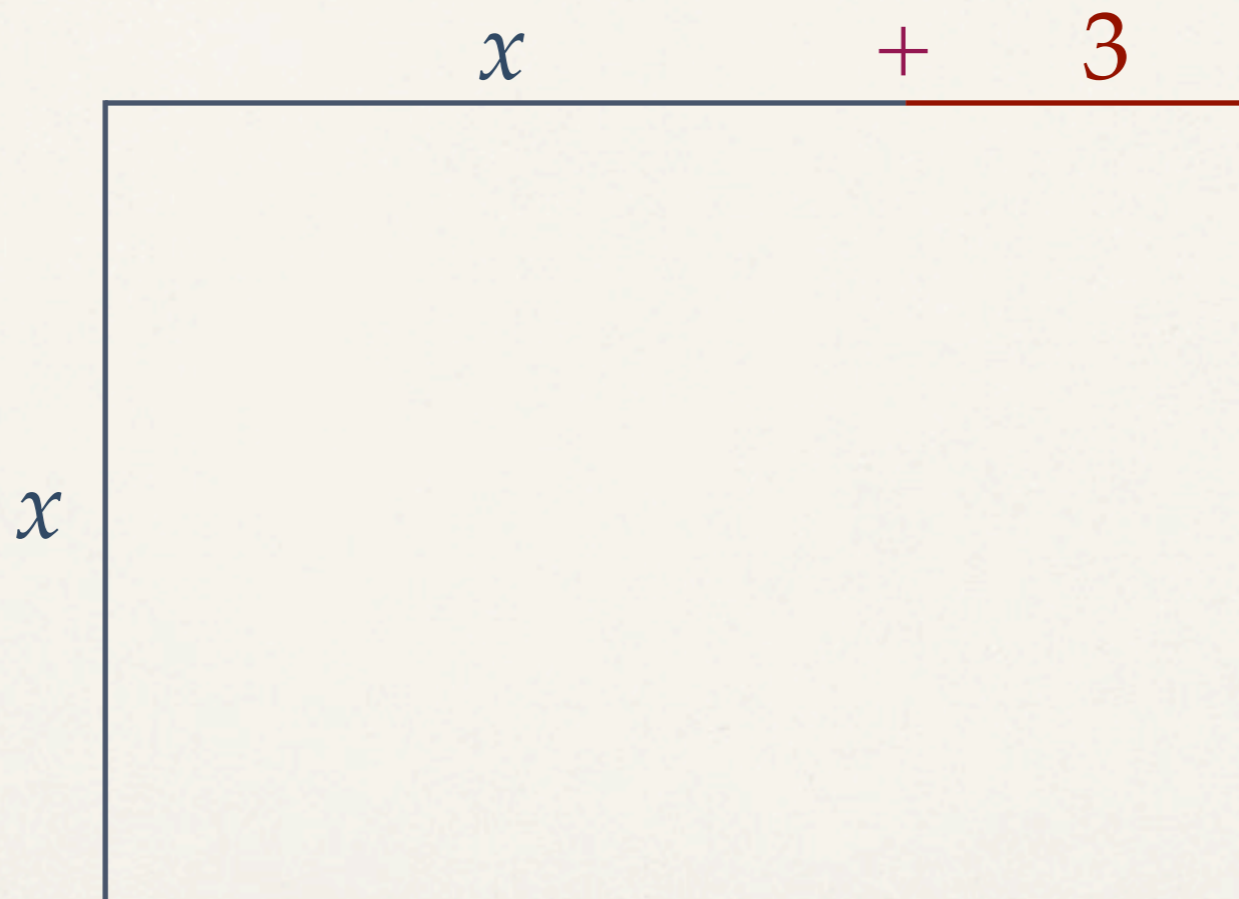
Presenting and representing

$$(x + 3)(x + 2)$$

$$\begin{array}{r} x + 3 \\ \hline \end{array}$$

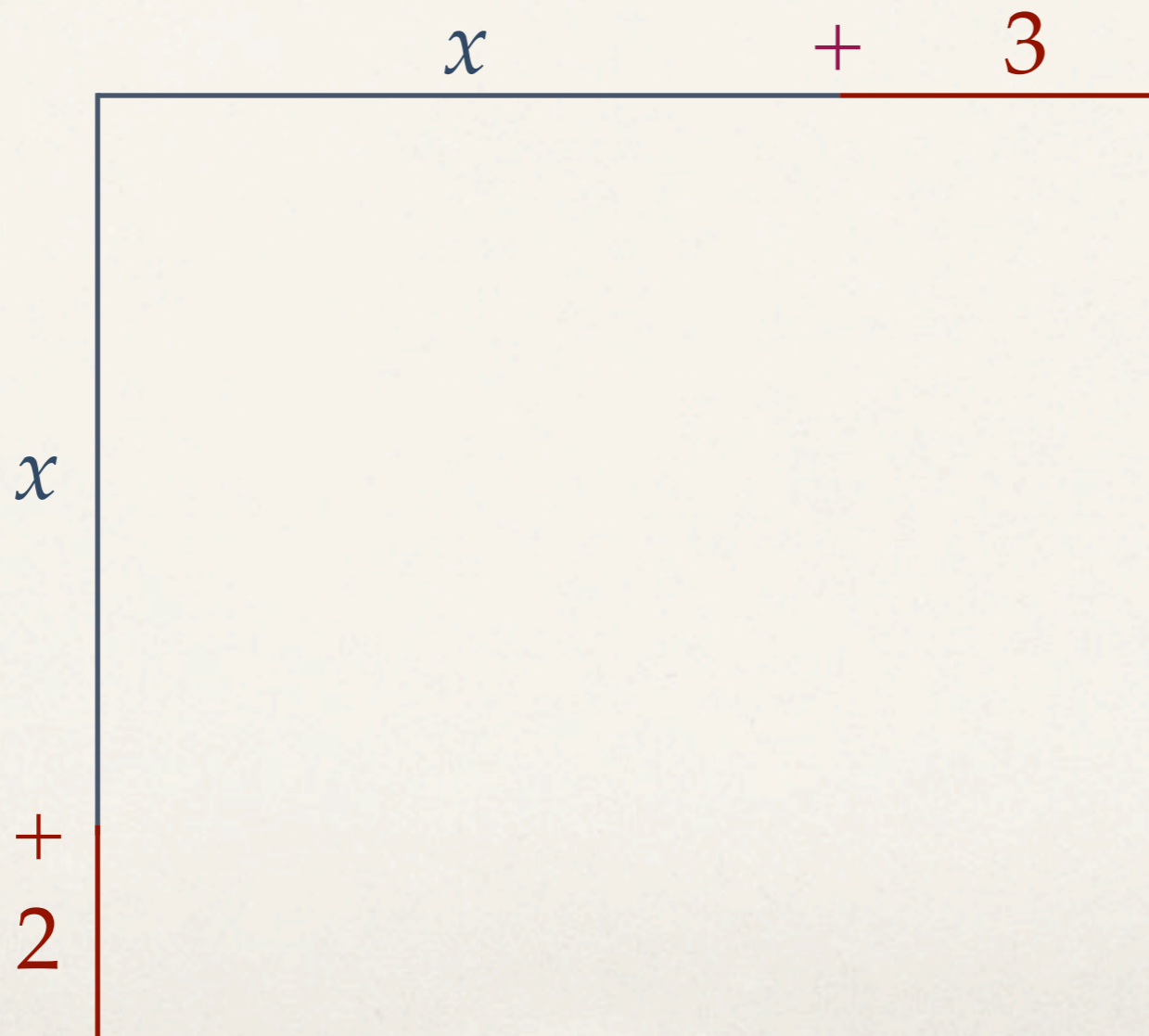
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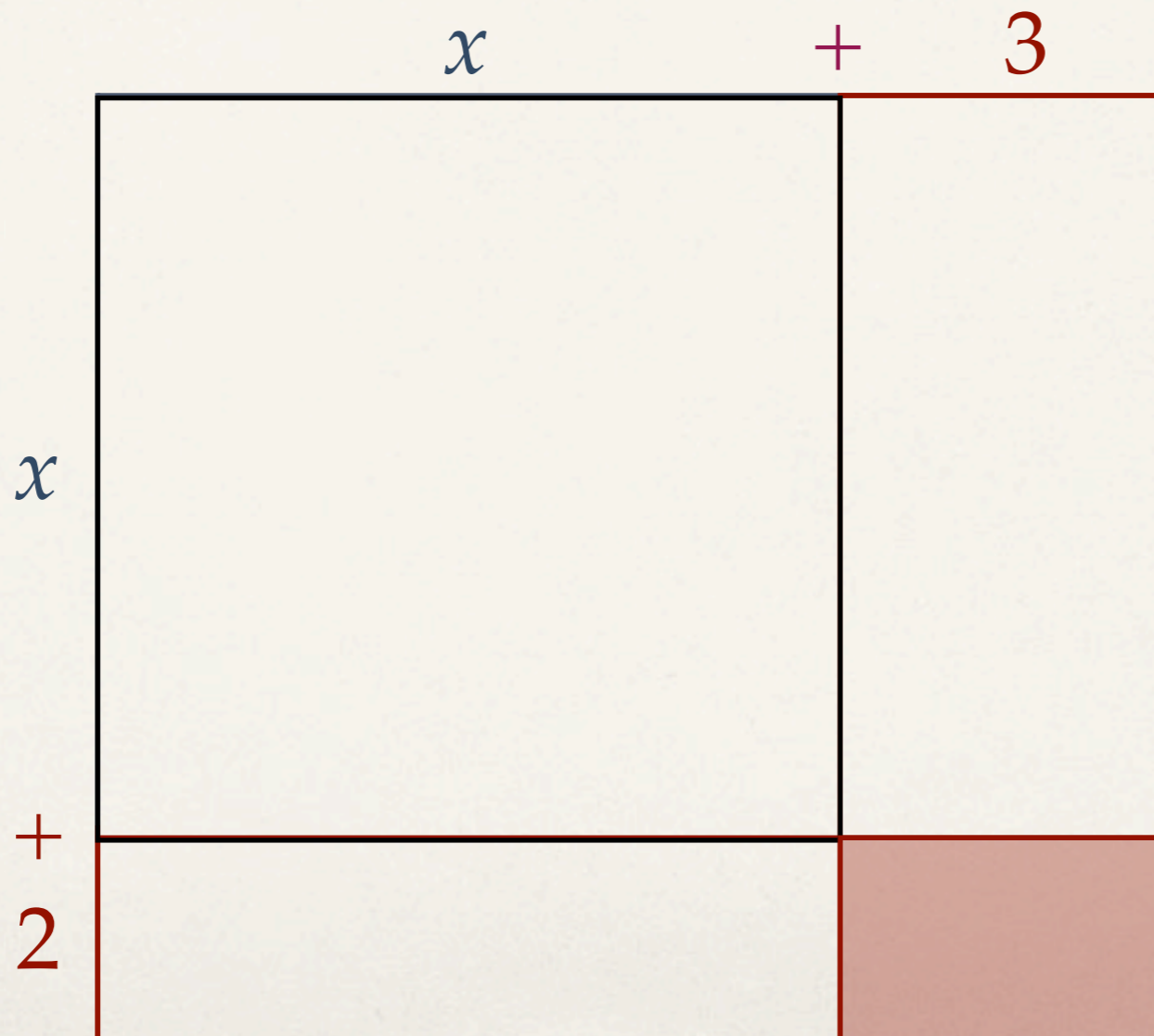
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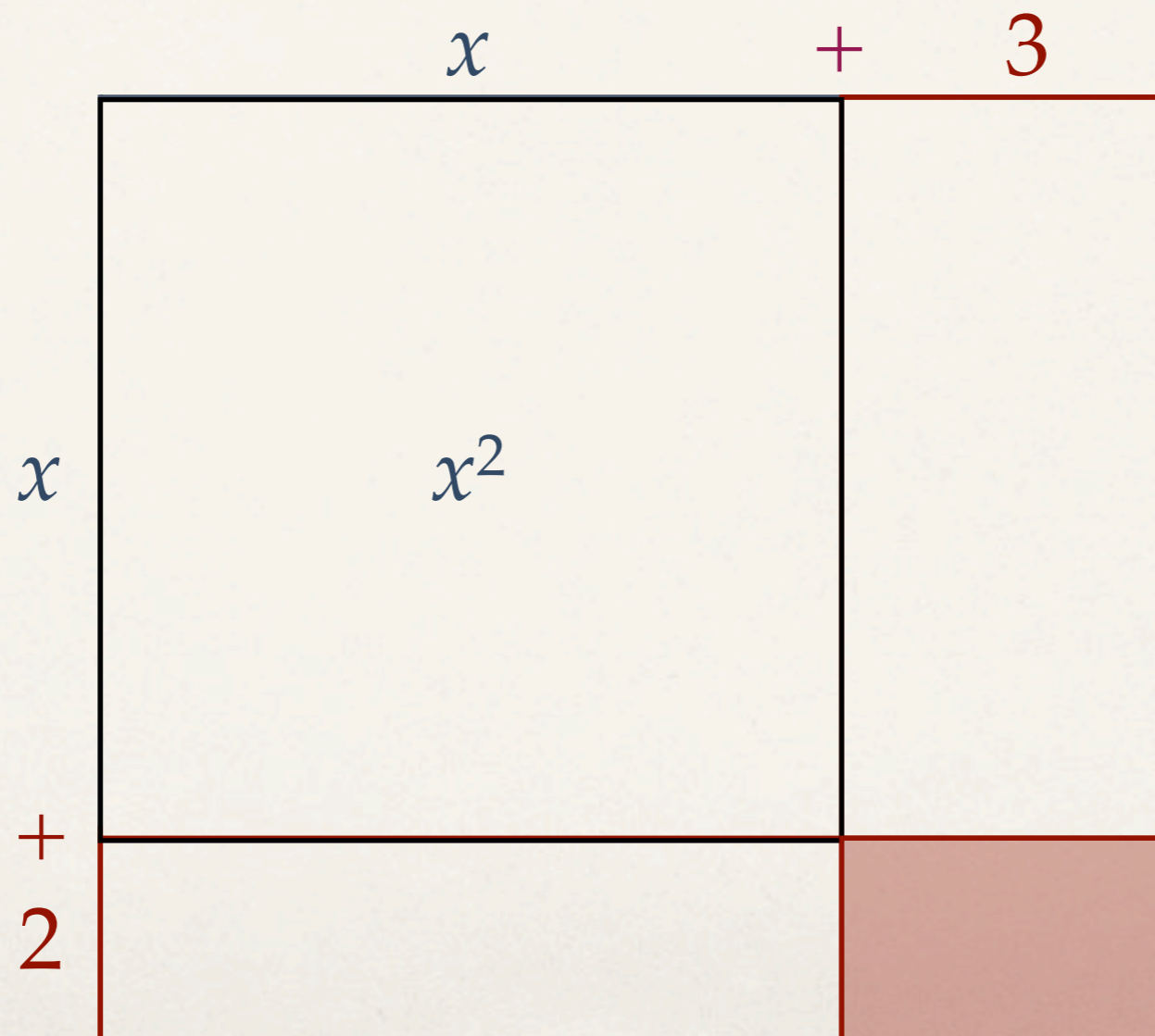
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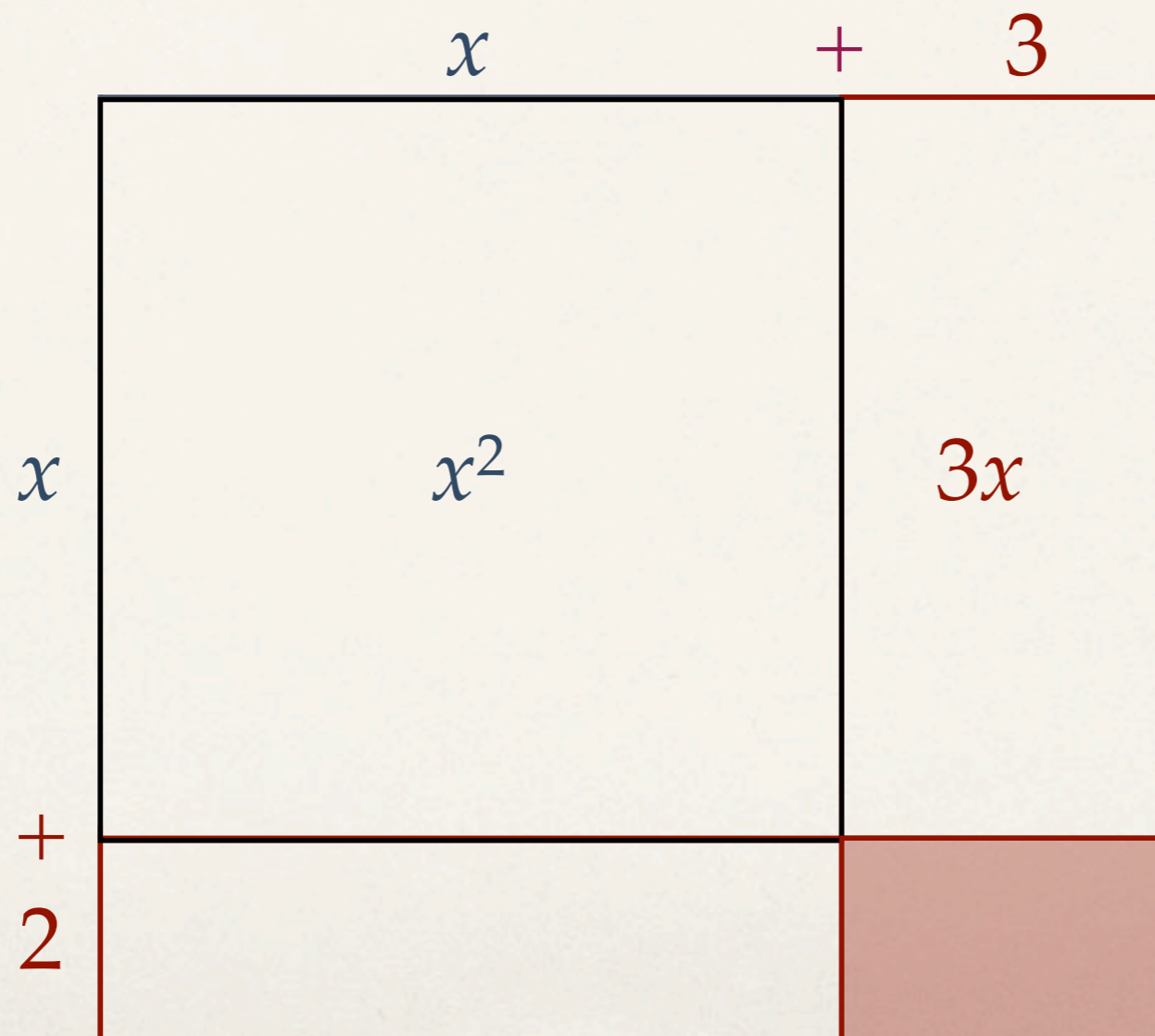
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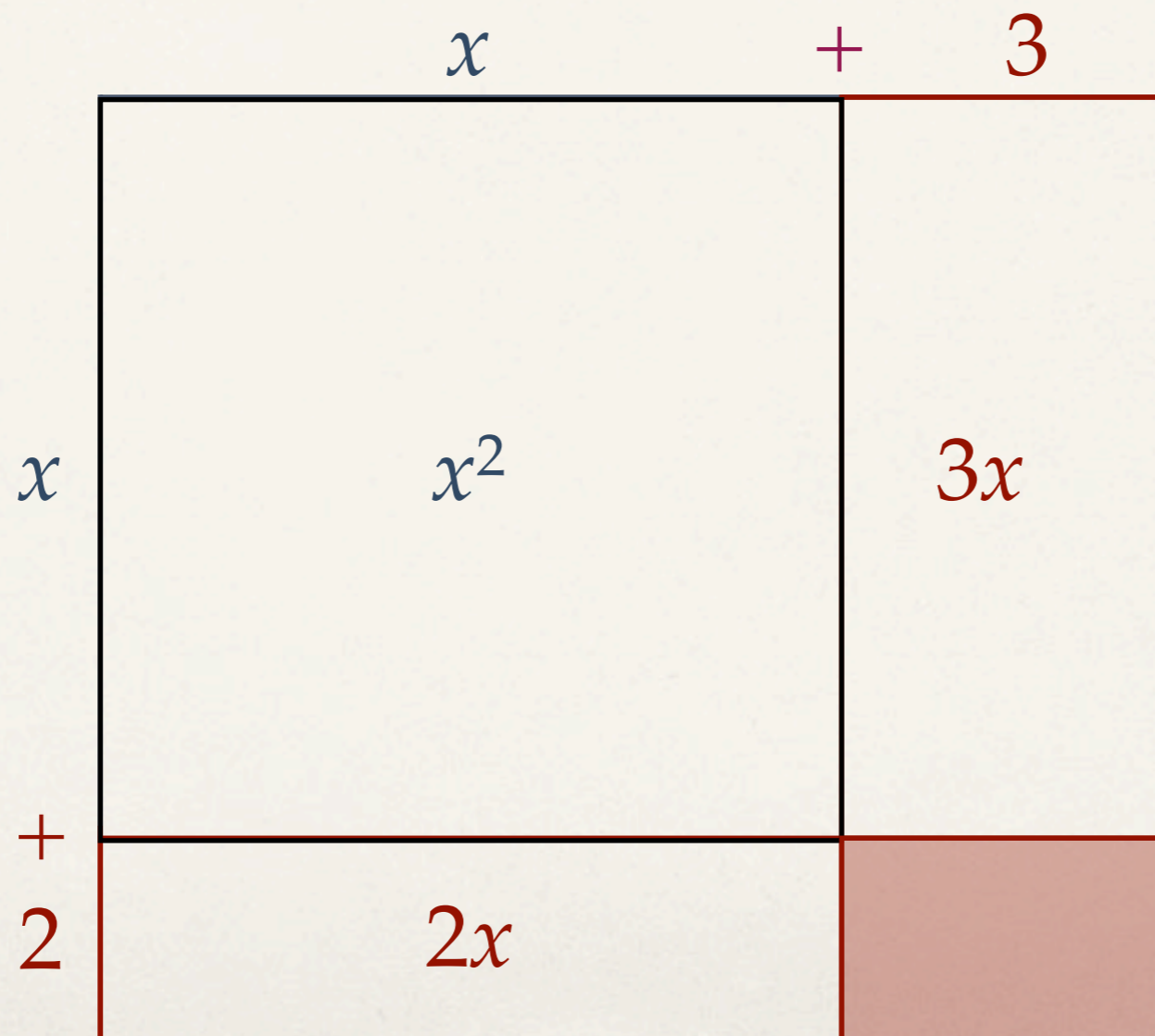
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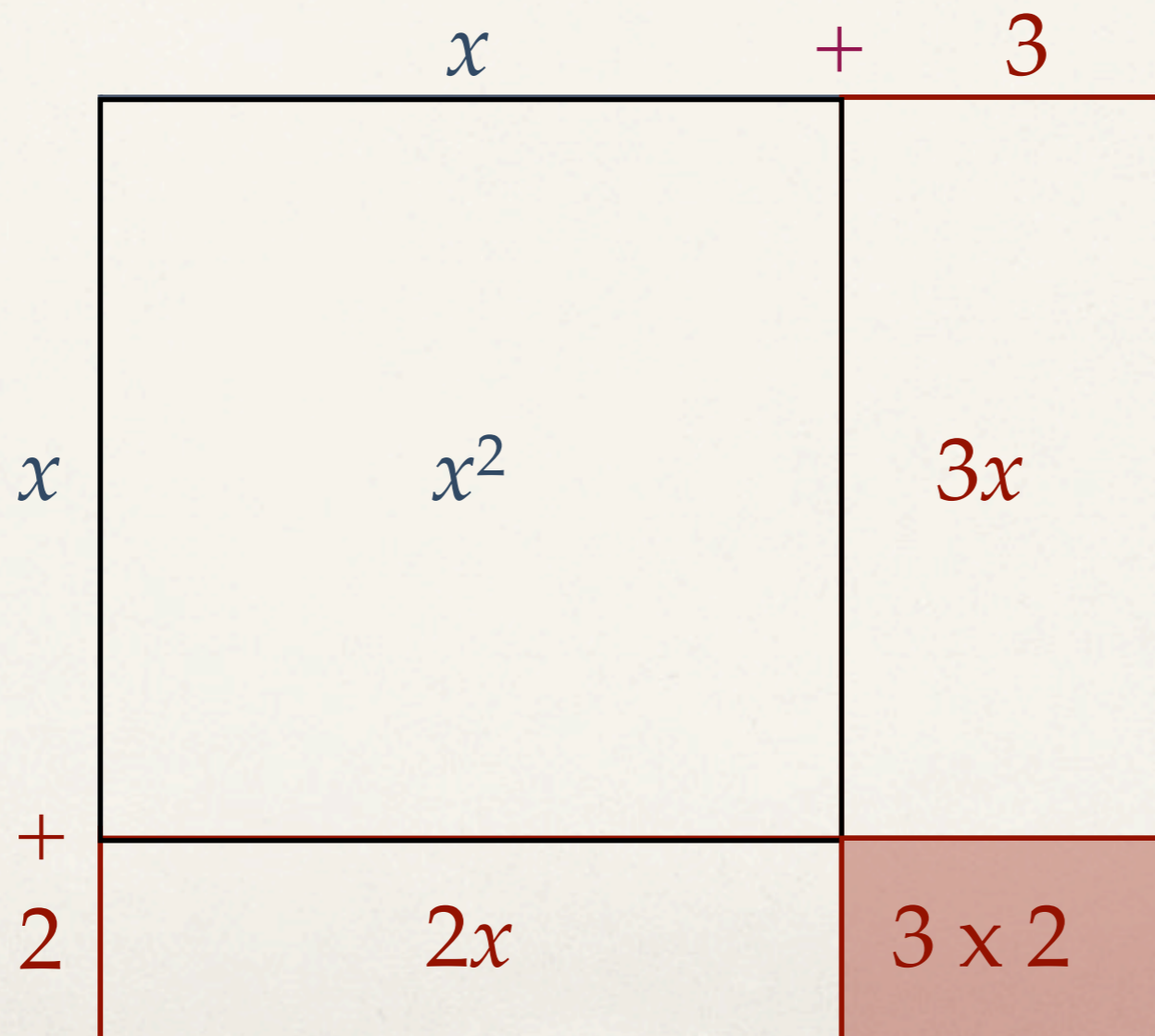
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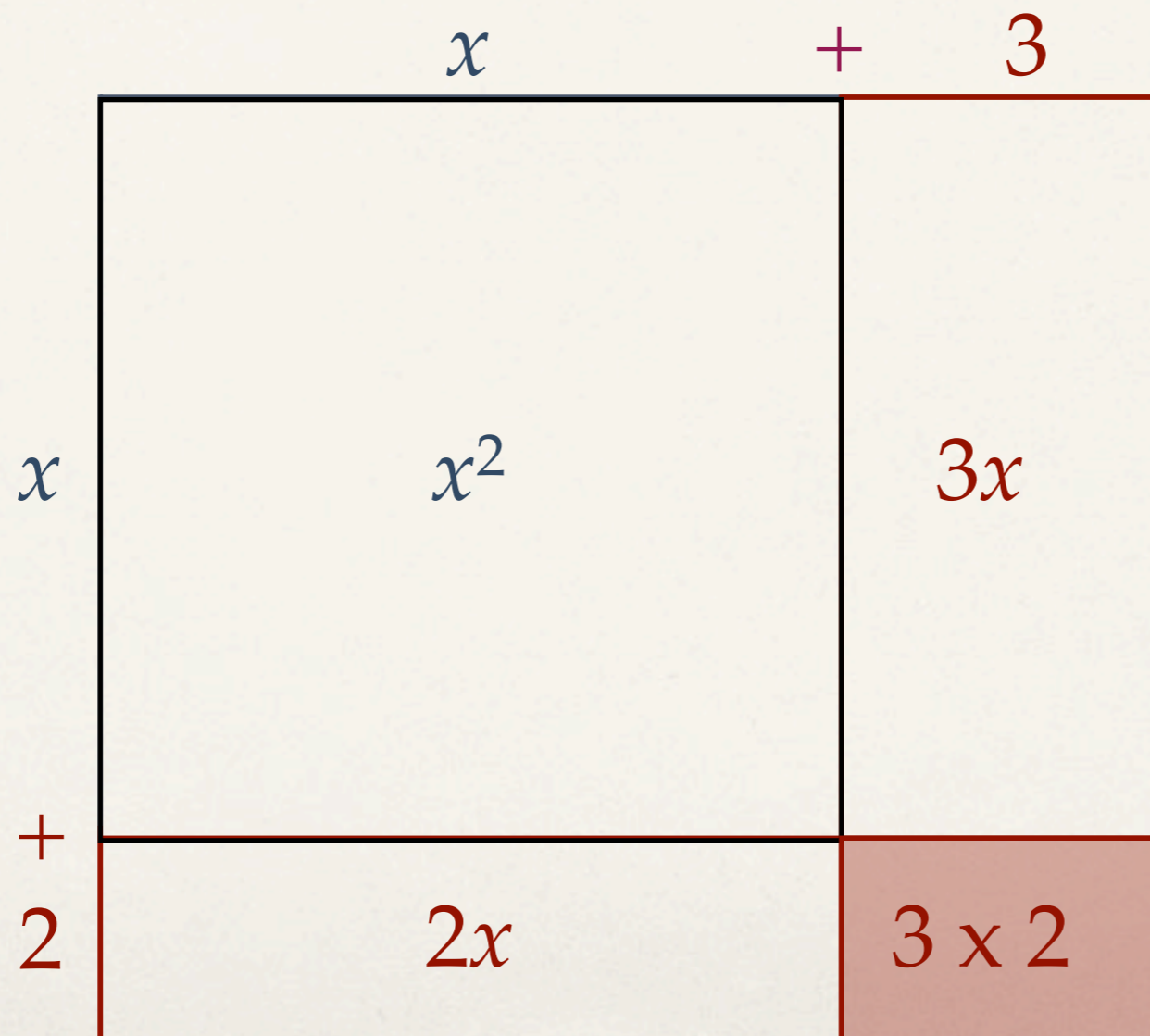
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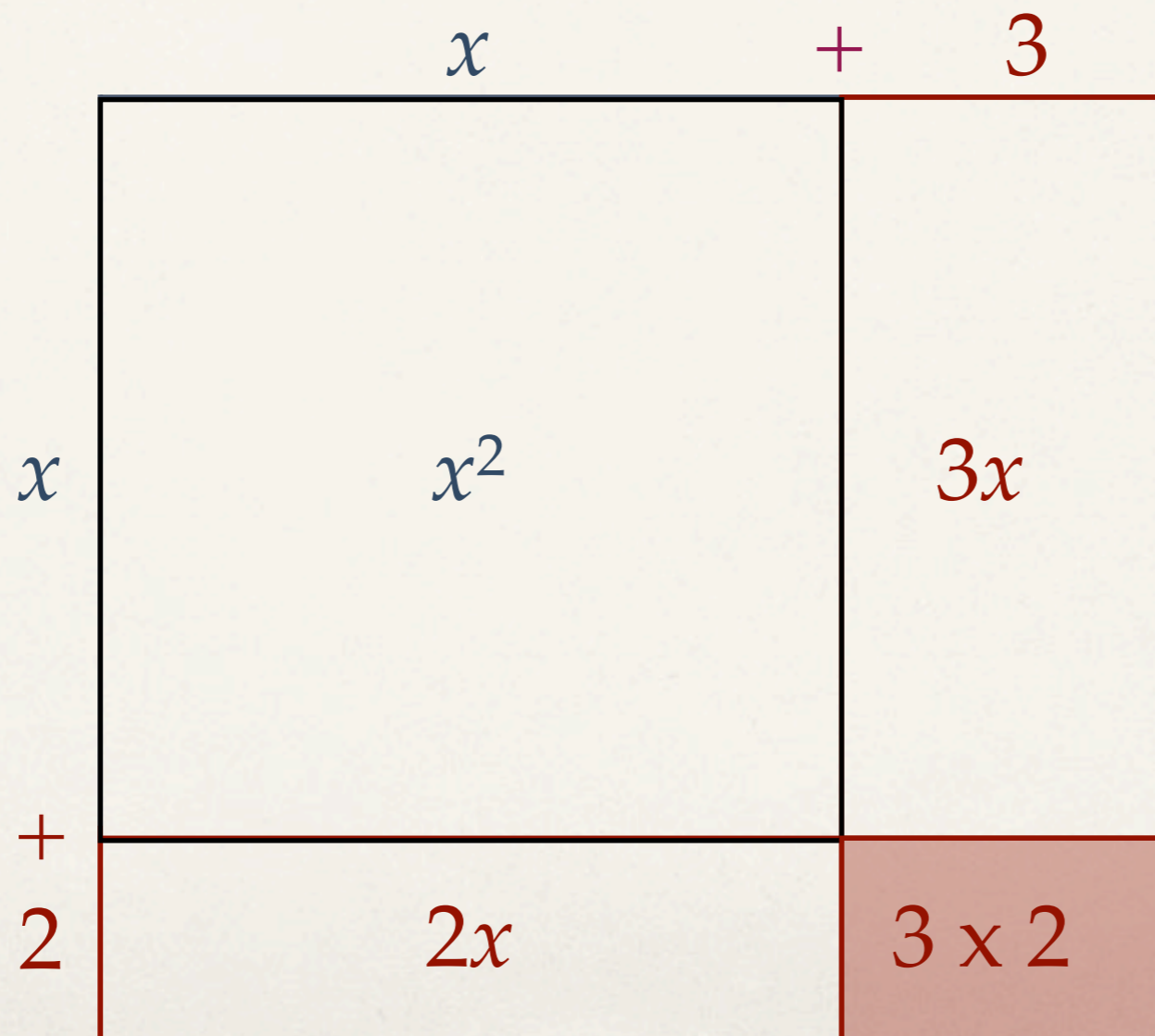
Presenting and representing

$$(x + 3)(x + 2) = x^2 + 3x + 2x + 6$$



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Presenting and representing

$$(x + 3)(x + 2) = x^2 + 3x + 2x + 6$$

	x	$+$	3
x	x^2		$3x$
$+$			
2	$2x$		3×2

Making the link:
Can you tell the difference between presenting and representing?

Presenting and representing

$$8 + 9 + 10 + 11 + \dots + 97 =$$

Presenting and representing

$$8 + 9 + 10 + 11 + \dots + 97 =$$

8



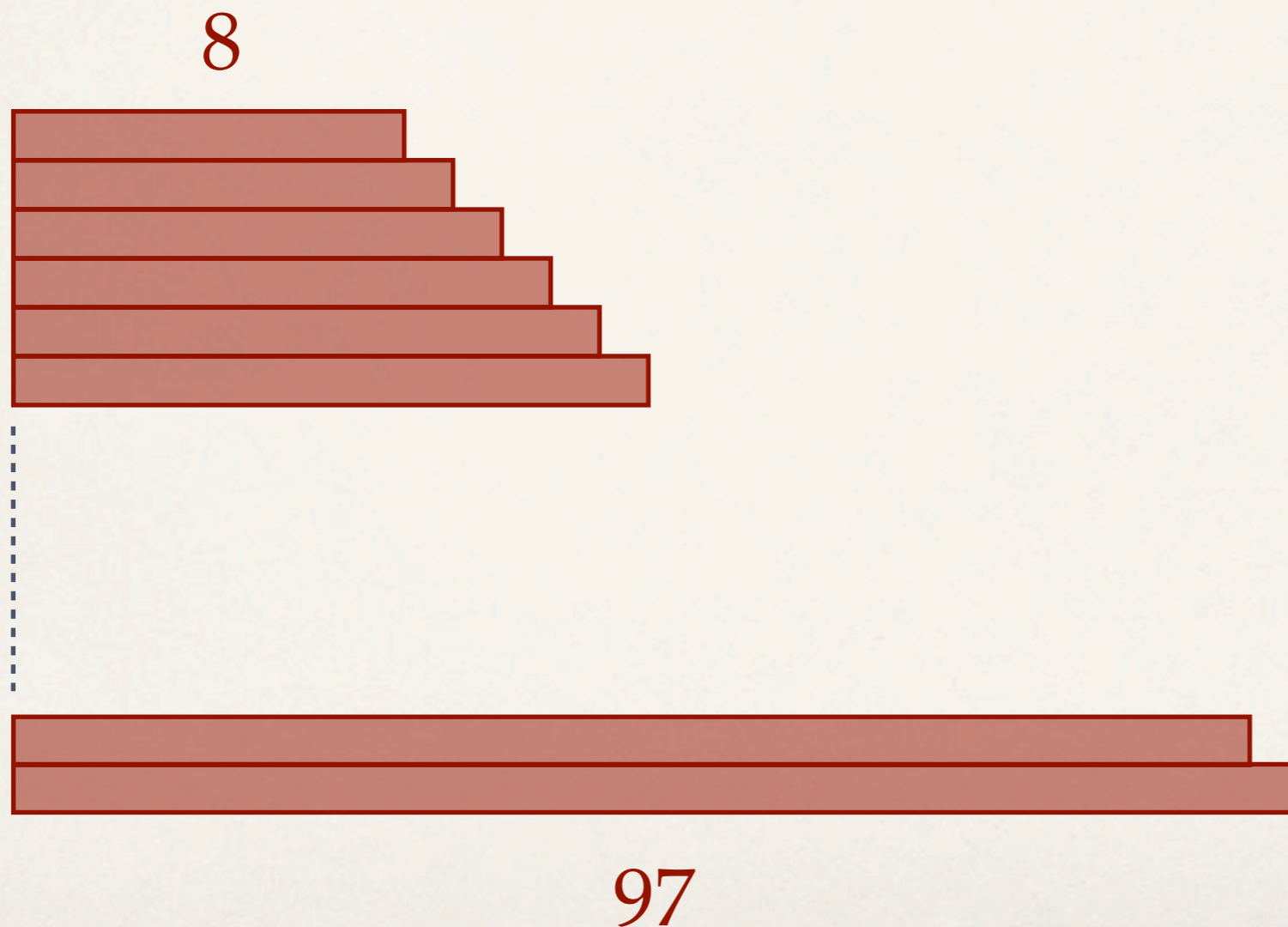
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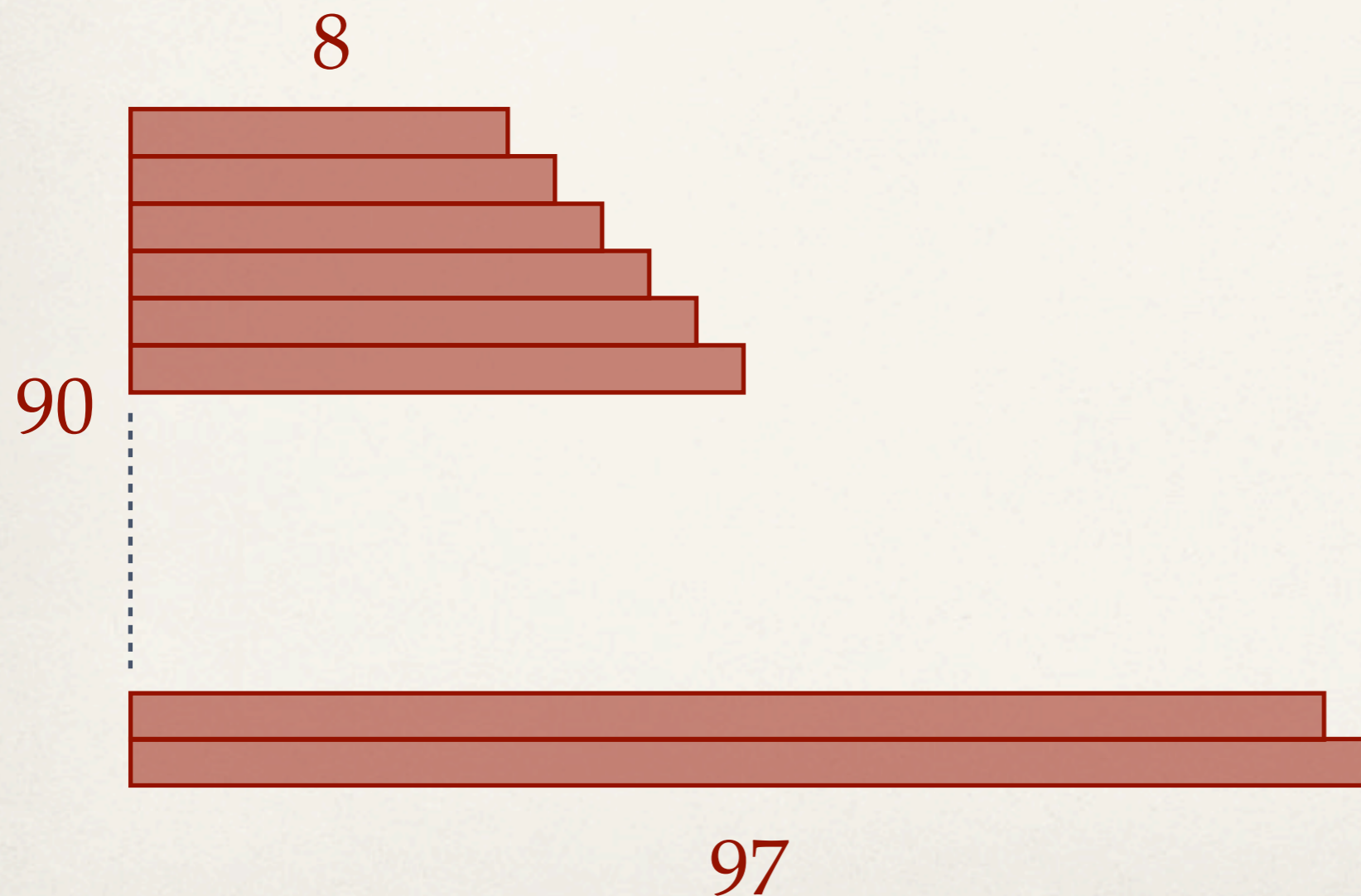
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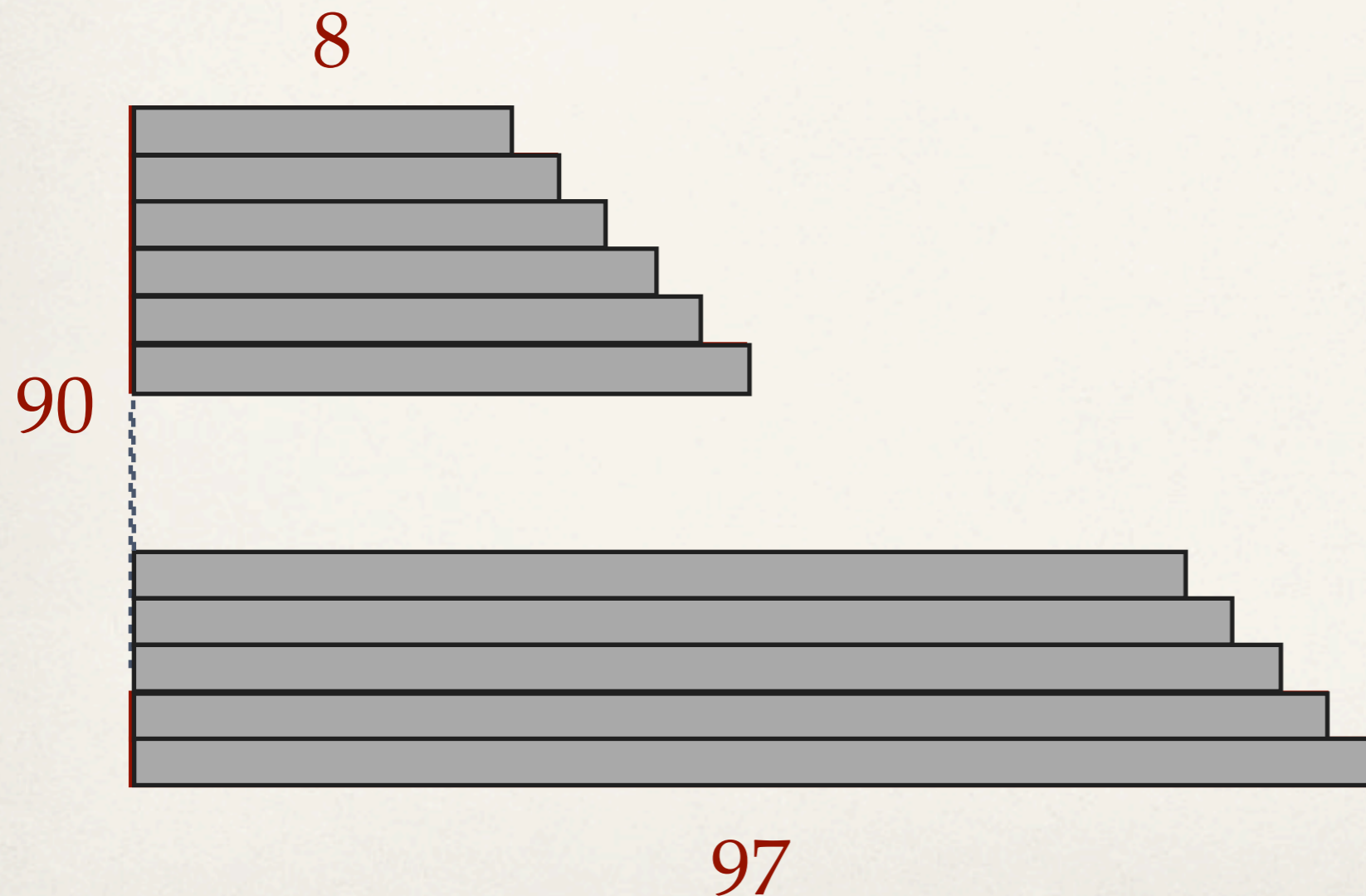
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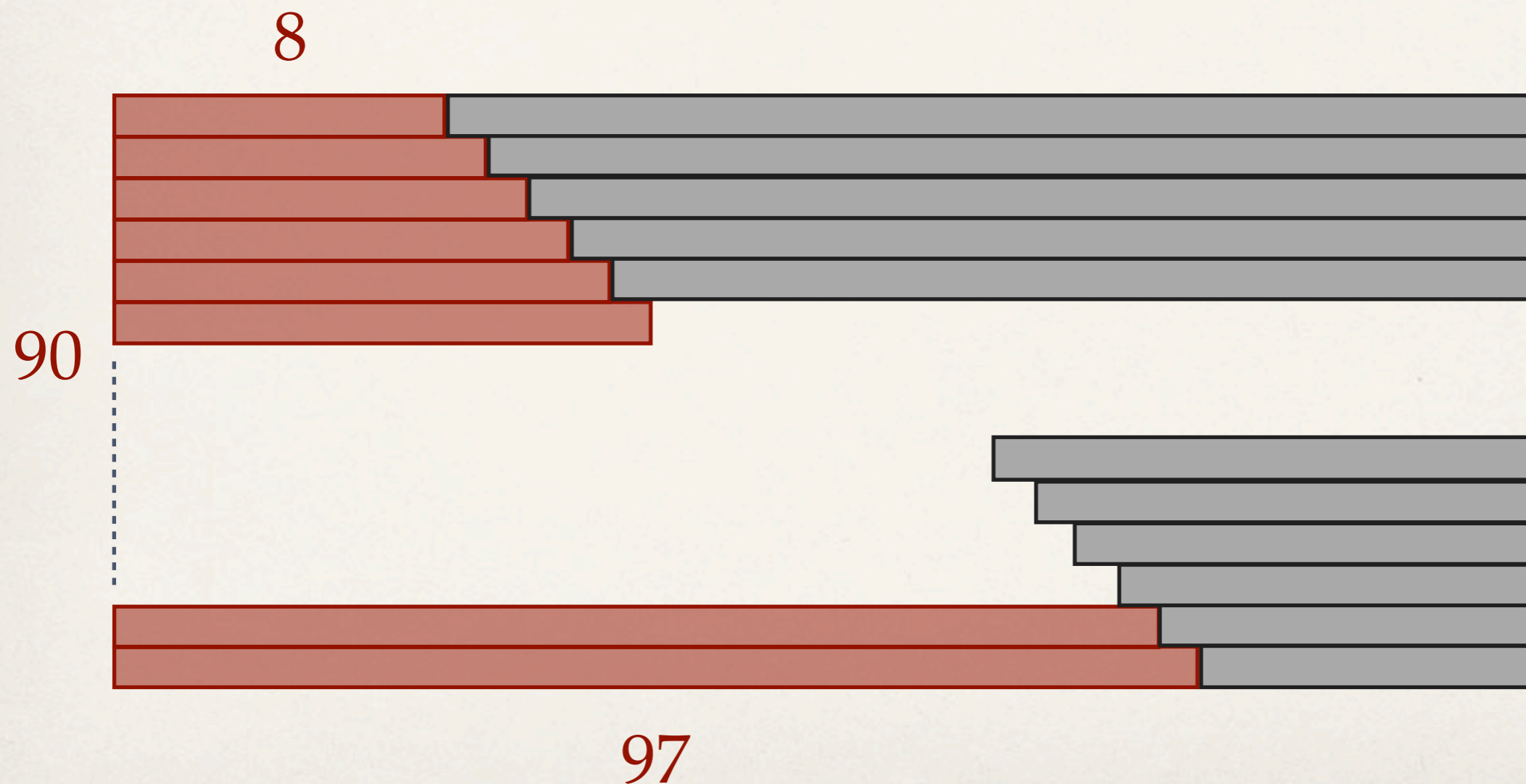
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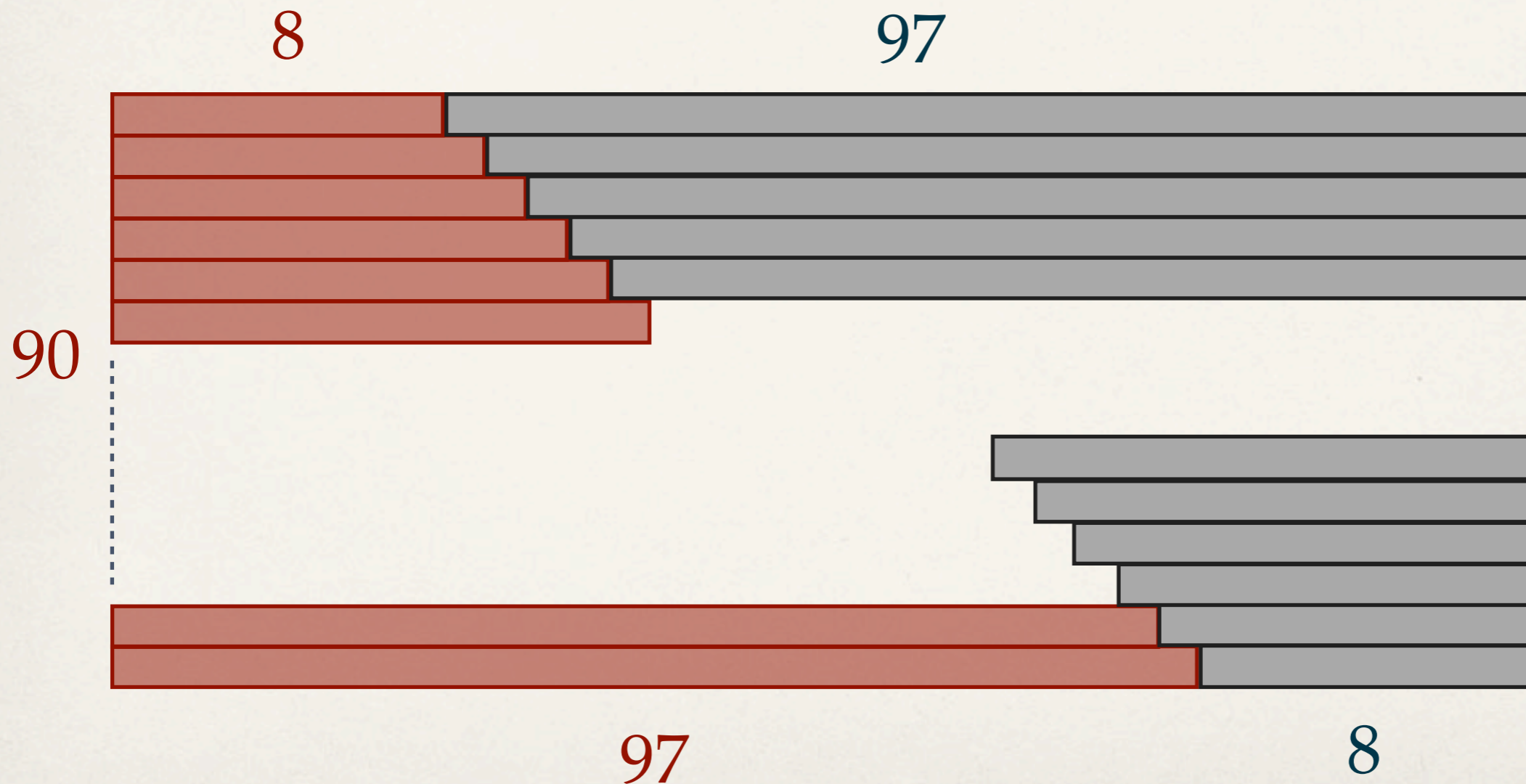
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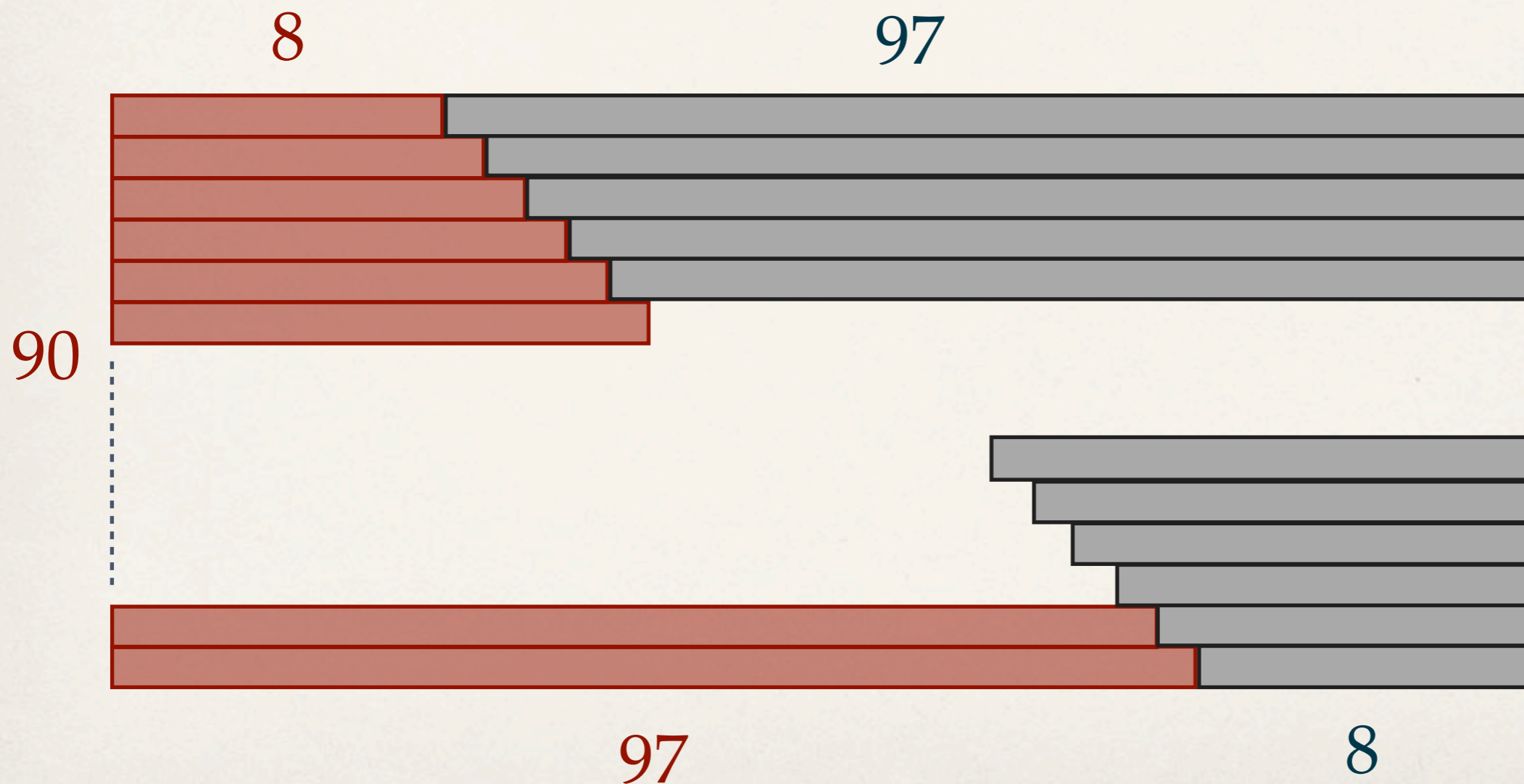
Presenting and representing

$$8 + 9 + 10 + 11 + \dots + 97 =$$



Presenting and representing

$$8 + 9 + 10 + 11 + \dots + 97 = \frac{1}{2} \times 90 \times (8 + 97)$$



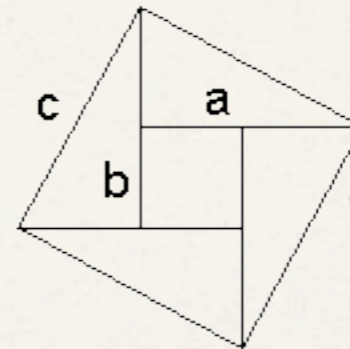
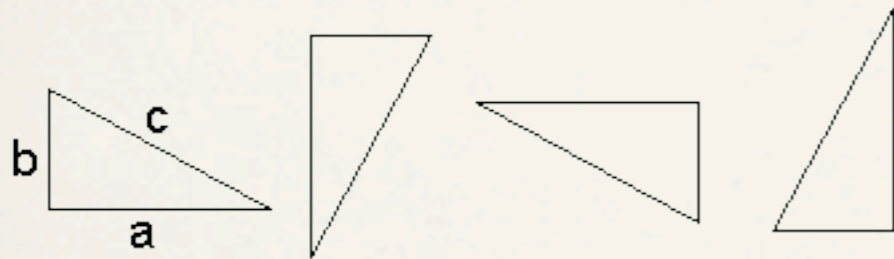
Are representations useful?

Are representations useful?

- ▶ Representations often provide the structure that is used in mathematical reasoning.

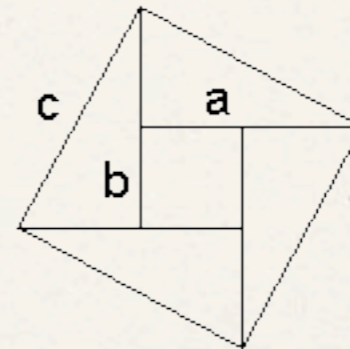
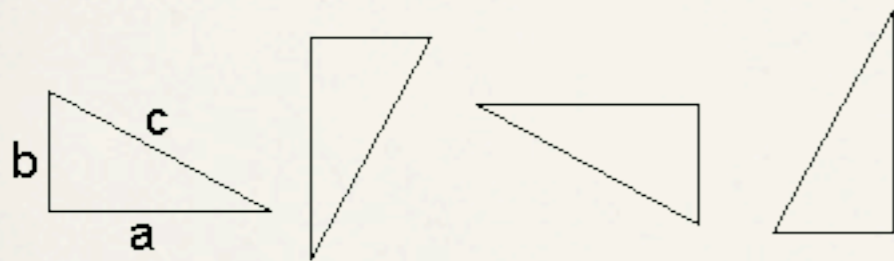
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The square has a square hole with the side $(a - b)$. Summing up its area $(a - b)^2$ and $2ab$, the area of the four triangles $(4 \cdot \frac{ab}{2})$, we get

$$\begin{aligned}c^2 &= (a - b)^2 + 2ab \\ &= a^2 - 2ab + b^2 + 2ab \\ &= a^2 + b^2\end{aligned}$$

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Q. The percentage of students who understand area well is:

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A. $> 90\%$

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B. $< 20\%$

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C. $\neq 100\%$

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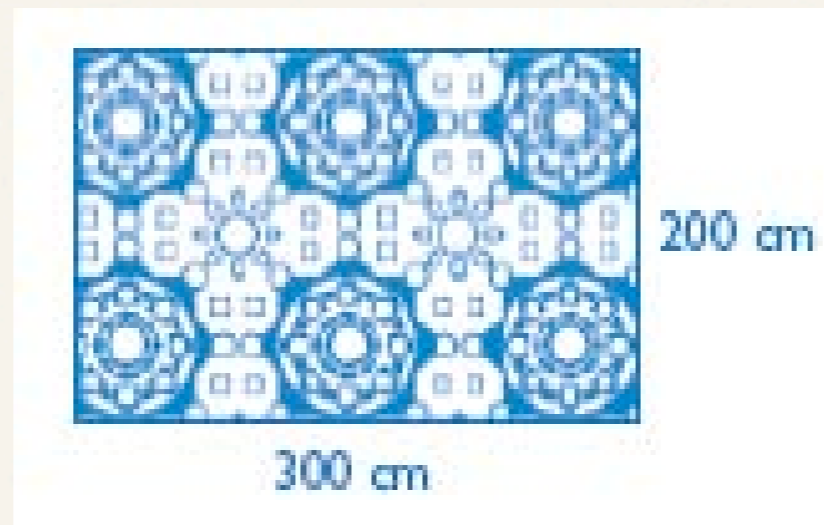
B. $< 20\%$

C. $\neq 100\%$

D. Don't know

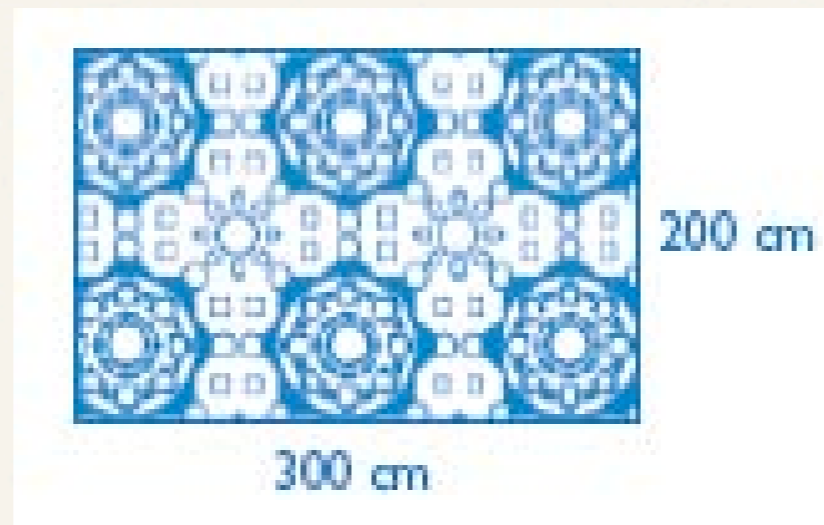
Roman tiles

During the Roman Empire some of the Romans put tiled panels in their villas.



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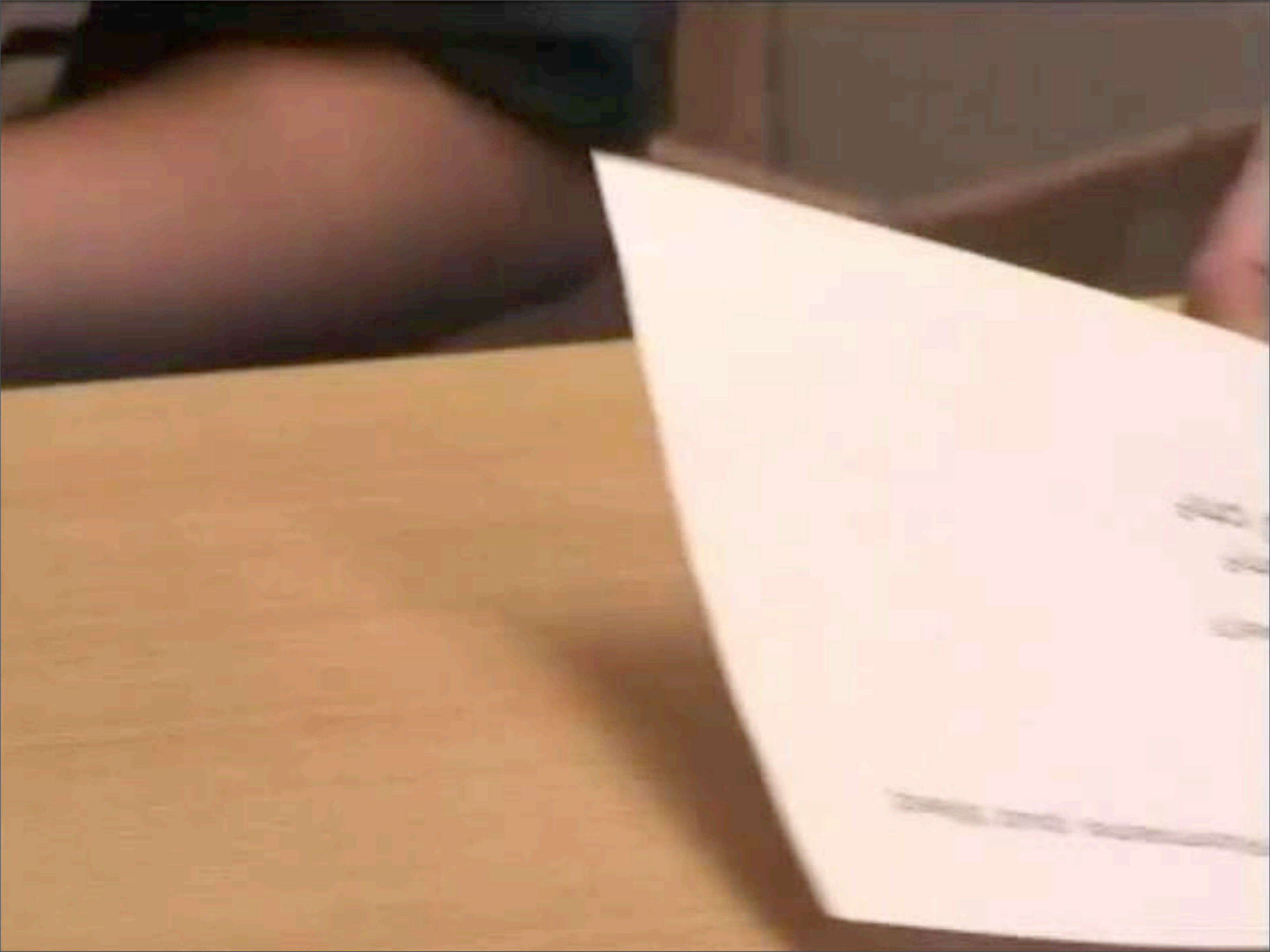
What is the area of this tiled panel?

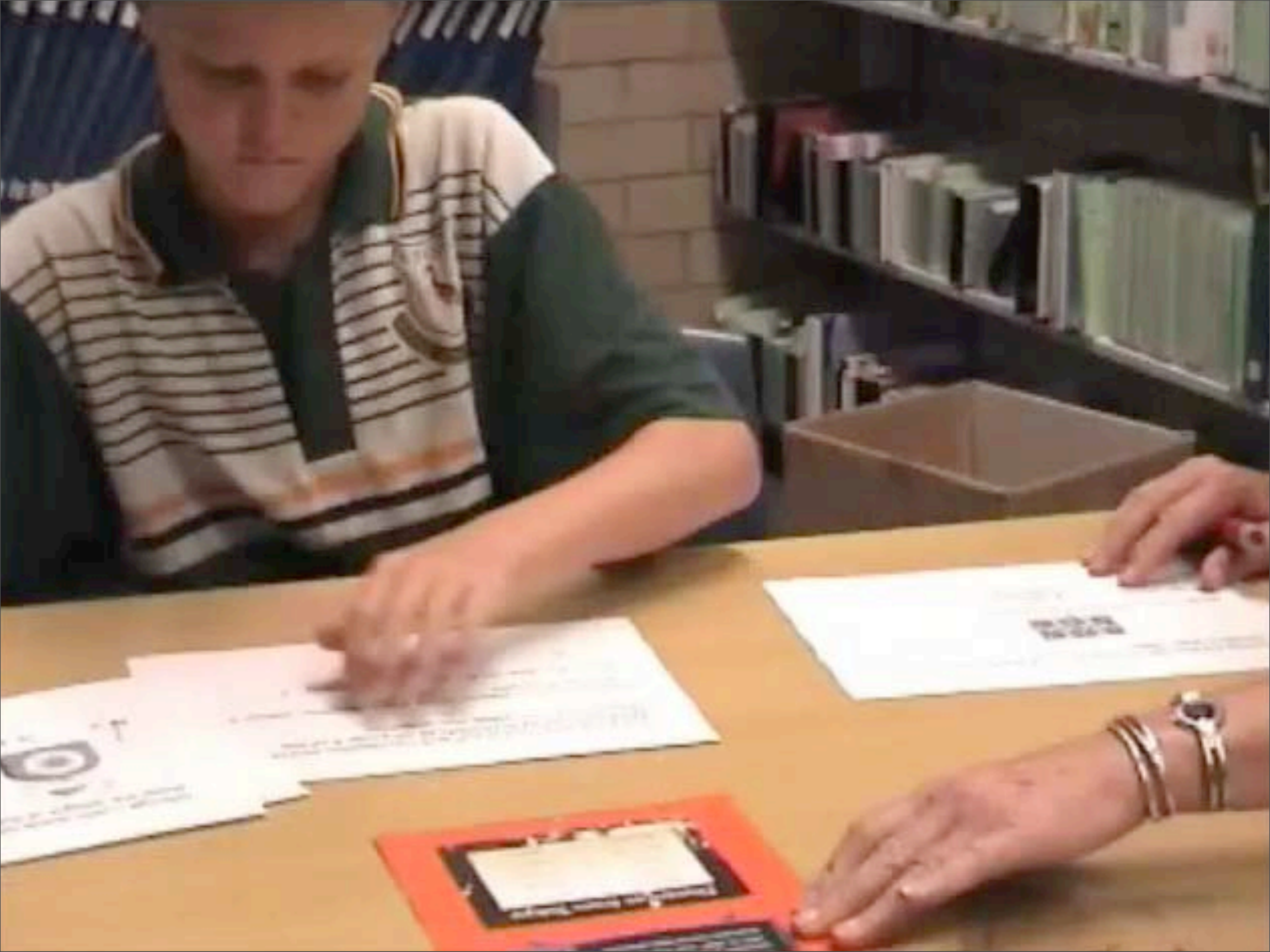
60 cm^2

6000 cm^2

600 cm^2

$60\,000 \text{ cm}^2$





A young man with short hair, wearing a striped polo shirt, is sitting at a wooden desk in a library. He is looking down at a piece of paper. In the foreground, another person's hands are visible, one holding a calculator. The desk is cluttered with several sheets of paper, some of which appear to be worksheets or assignments. In the background, there are bookshelves filled with books. The text "One-third of all Year 7 students chose 600 cm²." is overlaid in white serif font across the middle of the image.

One-third of all Year 7 students
chose 600 cm².

Our students wouldn't do that!

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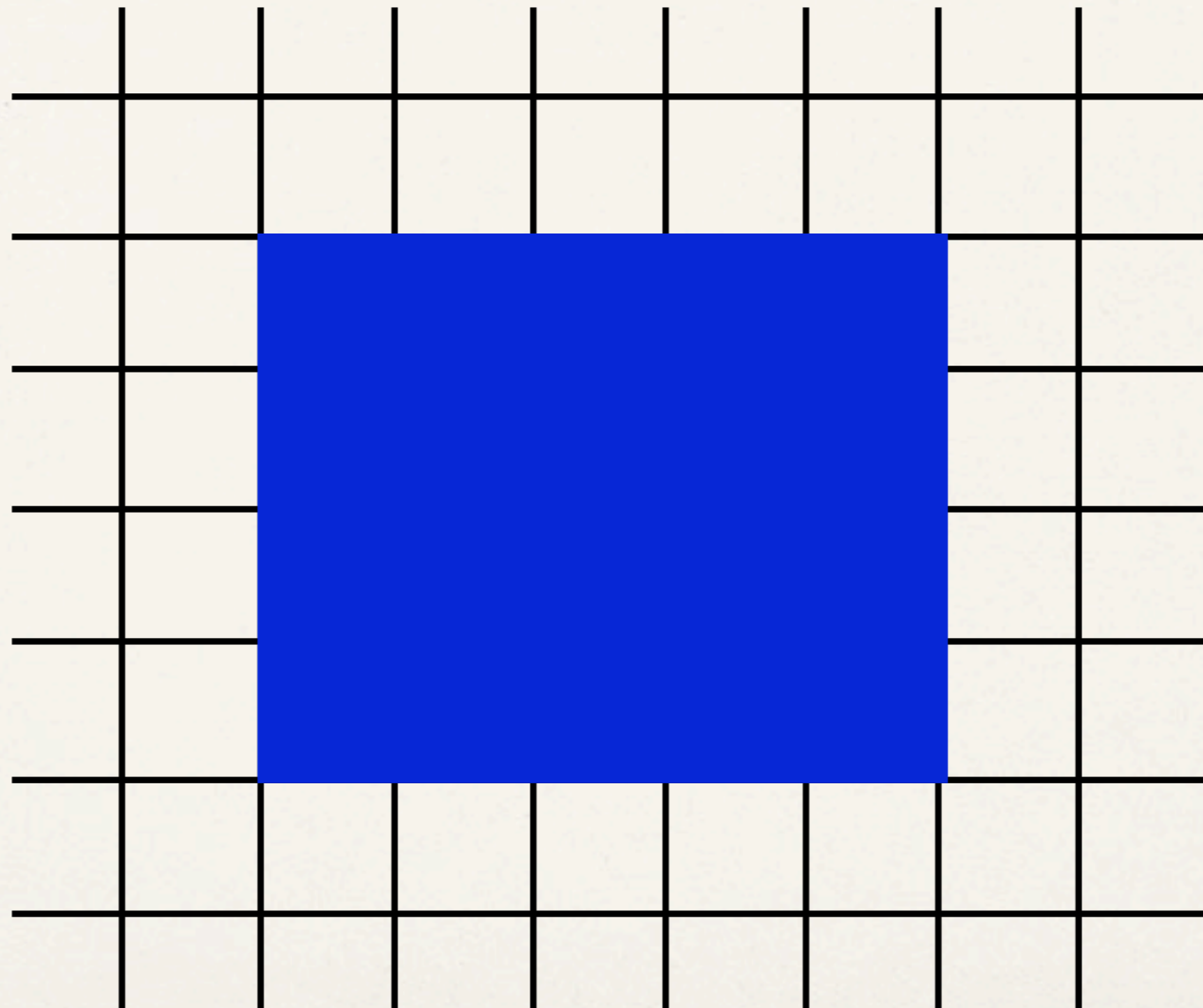
One-third of all Year 7 students chose 600 cm^2 as the area of a rectangle 300 cm by 200 cm. Some undoubtedly added the length and width and chose the closest answer.

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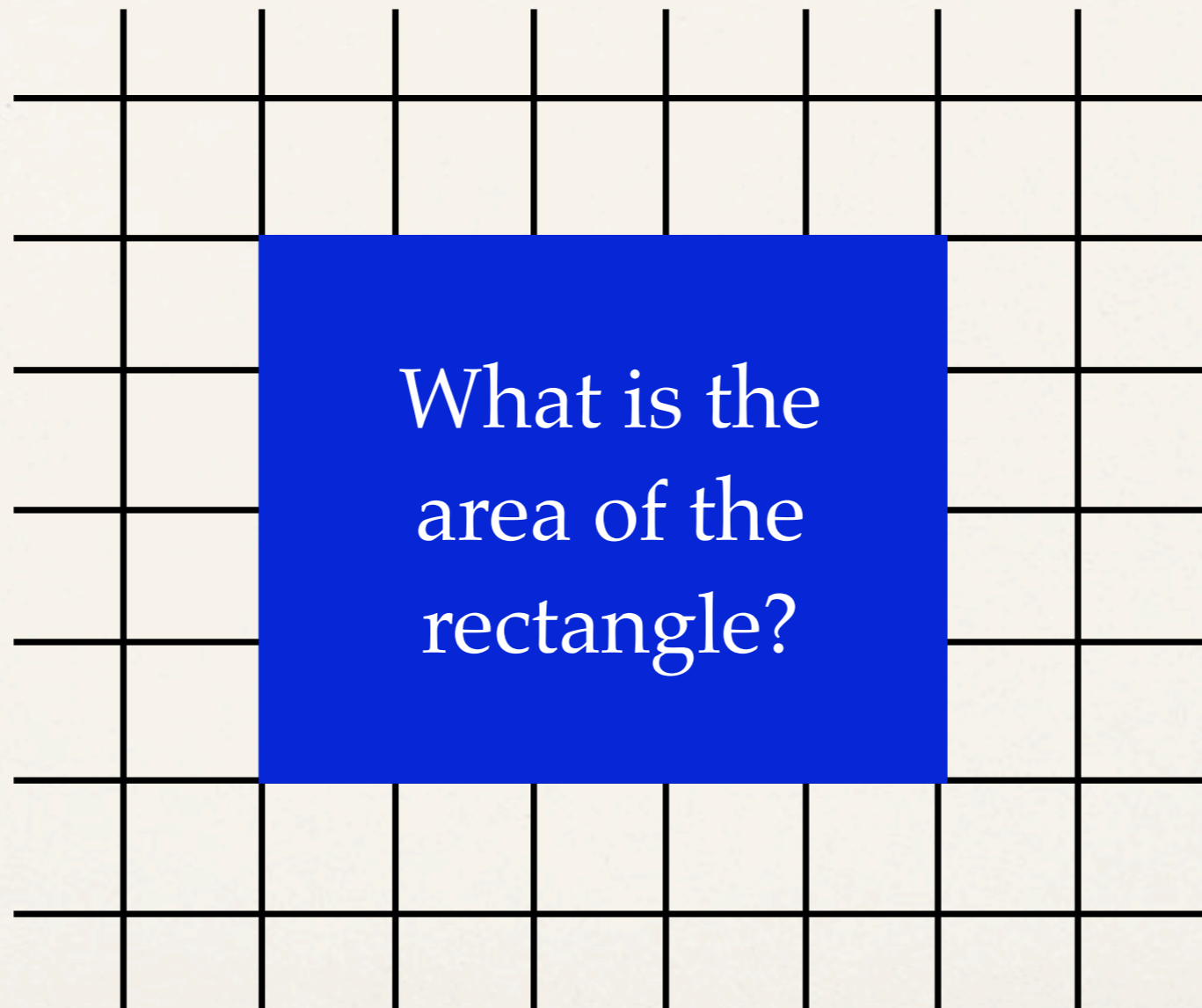
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How students represent the problem and reason with that representation does matter.

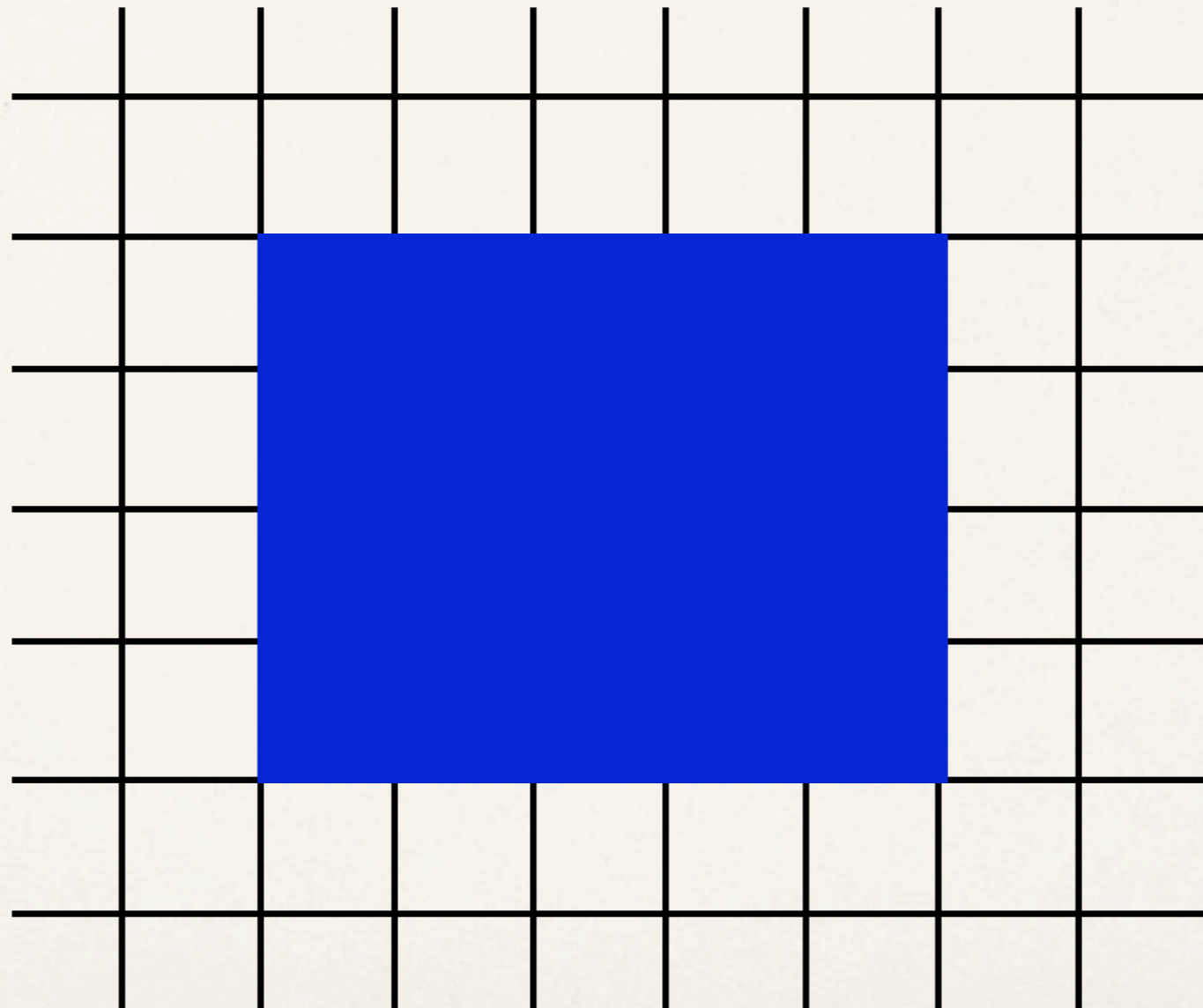
Representations and structure



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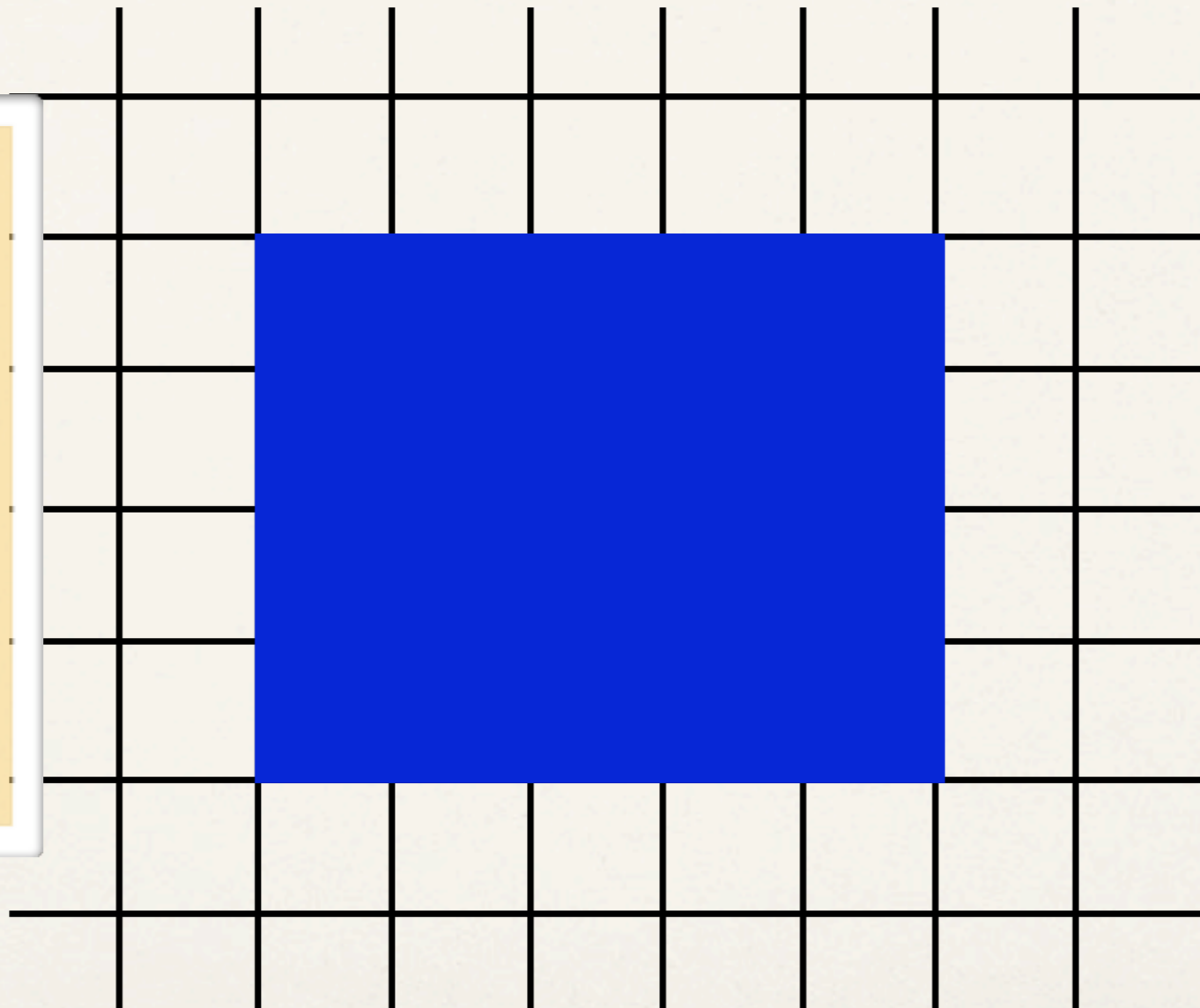


Using a background grid

Representations and structure

Don't they
just need to
know

$A = l \times b$ or
area is length
times width?



Using a background grid

From presenting to representing

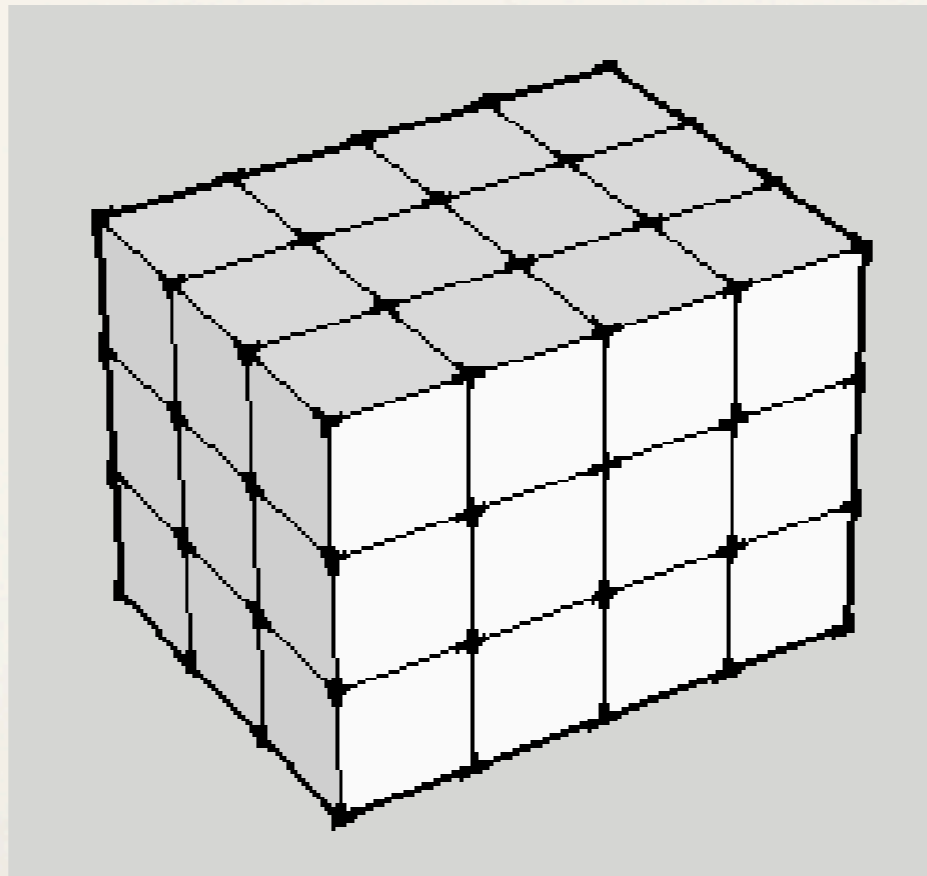
From presenting to representing

A teacher needs to be aware of multiple interpretations of materials so as to hear hints of those that students actually make.

From presenting to representing

Without this awareness it is easy to presume that students see what we intend they see, and communication between teacher and student can break down when students see something other than what we presume.

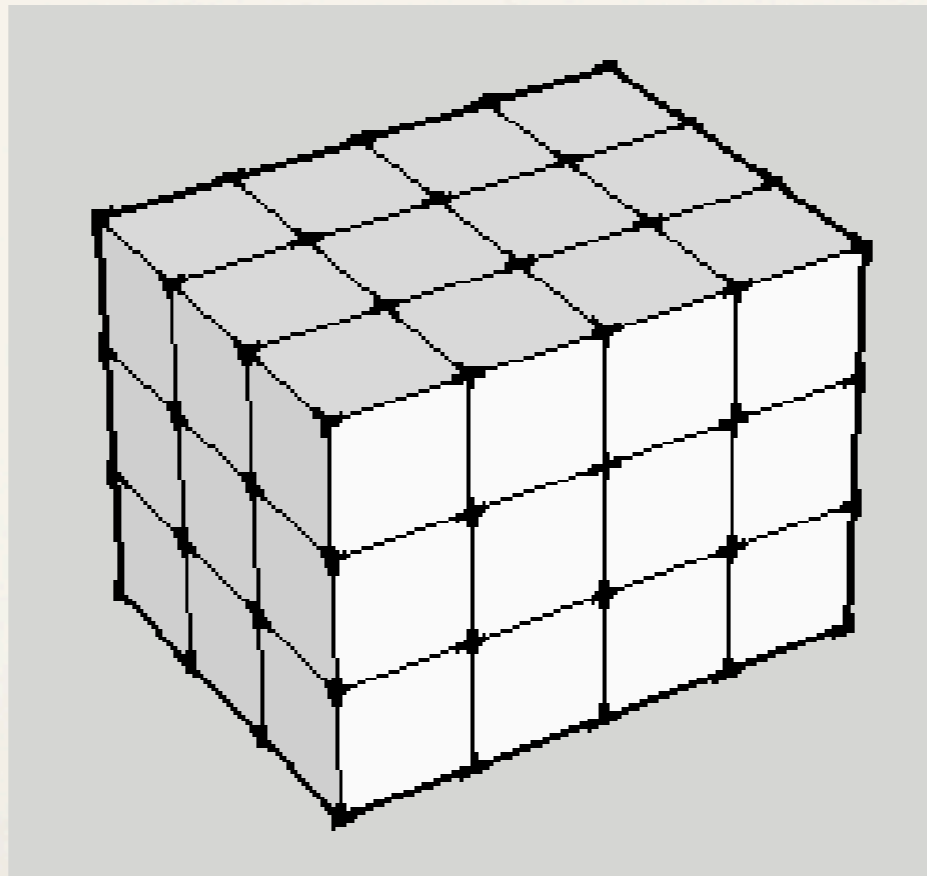
New tools for representing



New tools for representing

A rectangular solid is cut into cubes as shown.

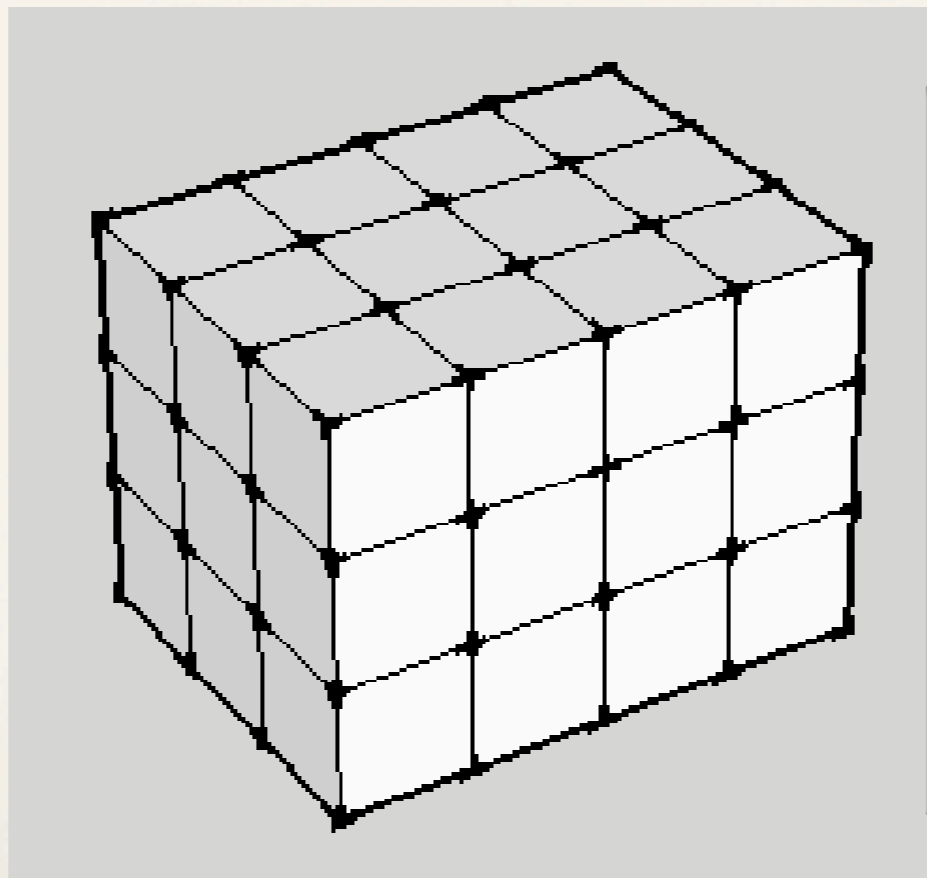
How many cubes are there?



New tools for representing

A rectangular solid is cut into cubes as shown.

How many cubes are there?



How many of
your Year 7
students could
answer this
question correctly?

New tools for representing

New tools for representing

In the 1977 National Assessment of Educational Progress in the United States, less than 25% of 13-year-olds and less than 40% of 17-year-olds were able to provide a correct answer.

The errors

The common errors made in answering this volume question include students:

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- counting only the cubes they can see,

The errors

The common errors made in answering this volume question include students:

- counting only the cubes they can see,
- counting only the faces they can see or,

The errors

The common errors made in answering this volume question include students:

- counting only the cubes they can see,
- counting only the faces they can see or,
- doing either of these processes followed by doubling the number.

Teaching ideas

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We could ask a student to make a model of the rectangular prism using cubes.

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However, to address the specific needs of this problem they would then have to create an isometric drawing of what they had made. Creating isometric drawings (even using dot paper) is quite difficult for many students.

Sketchup

Using new 3-dimensional drawing tools such as Google *Sketchup*, the drawing is formed at the same time as you create the object, and you can rotate the object to change your point of view.

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