The response of power systems to autonomous "grid friendly" devices

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1 Introduction

Transpower operates the New Zealand electric power grid. For this MISG project they were interested in the possible effects on the power grid of autonomous load controlling devices which are installed in domestic appliances (such as fridges, air conditioners, heaters). These devices are known as grid friendly devices as they have the potential to assist in the management of grid disturbances by reducing load when there is a drop in power supply (U.S. patent 4317049, "Frequency Adaptive Power-Energy Re-scheduler", "Grid Friendly" is a trademark of Battelle Memorial Institute). The grid friendly devices are implemented as a small electronic control unit in domestic appliances such as water heaters and refrigerators.

In an alternating current grid the system frequency drops from the normal 50 Hertz to a lower value when the demand for power (load) is greater than the power being generated. A sudden disturbance, such as a generator going offline, causes a drop in frequency until the control systems increase power generation to make up for the loss. A typical event causes the frequency to drop for about a minute. Grid friendly devices use this connection between load and frequency to determine when to turn appliances off and then back on.

The grid friendly devices are capable of reducing power consumption more rapidly than generation can be increased, and hence can make the power distribution grid more stable and allow quicker recovery from disturbances. This is important as a large uncorrected disturbance can cause load shedding and the protective shutdown of equipment, with resulting major power outages. However unlike other protection devices (such as controlled load shedding) on the power distribution grid, the grid friendly devices work autonomously and are not under the control of the grid operator. One concern is, since not all appliances will be using power when a disturbance occurs their effect is variable, and hence the grid operator cannot predetermine the size of the effect. The grid friendly devices also effect the power balance on the grid when they switch back on, and again this is not under the control of the grid operators. Under some circumstances the effect of the grid friendly devices may be too large and make the disturbance worse.

The aim of this study was to examine whether the grid friendly devices can have undesirable effects on the power grid, and what design of grid friendly device is best suited to providing additional protection to the grid. This report first develops a simplified model of a power grid, looks at the conditions for stable operation of grid friendly devices, examines alternative designs, and makes recommendations for the design of grid friendly devices.

2 Previous investigations

The Pacific Northwest National Laboratory has conducted a demonstration project [1] in several communities in Oregon and Washington states. This project involved 50 retrofitted residential water heaters, 150 specially manufactured clothes driers, and ran for one year. As the study was conducted in the U.S.A, the nominal frequency is 60 Hertz, as opposed to the 50 Hertz used in New Zealand. The controllers switched off heating elements if the frequency dropped below 59.95 Hertz and switched the heating elements on again 16 seconds after the frequency rose above 59.96 Hertz. The controllers were connected via the internet to a central recording system, to assist in project evaluation. This project found that once initial engineering and approval problems were overcome, the equipment ran smoothly with the users not generally noticing any difference. Some problems occurred with data recording, however it was concluded that the project had successfully demonstrated feasibility and the potential to make a significant contribution to the stability of a power grid. It was estimated that a reserve of up to 20% of the total power usage could be made available through the use of grid friendly devices.

Other investigations of the use of grid friendly devices have generally been restricted to simulation studies. Lu and Hammerstrom [2] undertook a preliminary study for the demonstration project. For this study they collected data on the distribution of grid frequency over two years and used this to determine switching frequencies for the grid friendly devices. The switching frequency was chosen to act before the substation frequency protection relays would activate. These protection relays reduce load by creating blackouts. A plot of the number of occurrences and their duration for different switching frequencies was produced. The switching frequency used was chosen to give a reasonable number of switching events for the demonstration project. They mention but did not investigate the possibility of randomising the switching frequency and the delay time.

Recent papers report other simulation studies [3, 5] that include the effect of switching refrigerators off depending on both the grid frequency and the temperature within the refrigerator. For these units the switch off frequency is increased if the refrigerator temperature is sufficiently low. [5] also included a similar scheme for heaters in their analysis. [3] showed that in the presence of significant wind power (which is inherently variable) on the grid, the grid friendly devices reduced the variation in frequency and extended the time available to bring in more generation capacity. Both papers concluded that grid friendly devices would have significant benefits.

The literature has identified significant potential benefits from the grid friendly devices. However it is not clear what the effect of switching large loads, that are a significant portion of the total load, might be on the stability of the grid. This report looks at the effects of different designs, on both the frequency stability of the grid, and at the effect on the controlled generation that is needed to compensate for changes in the power balance on the grid.

3 Transpower model of a power grid

A power grid consists of generators that provide power, consumers that use power and the power lines that connect generators to consumers. As there are both multiple generators and many power users, the grid is a complex network of interacting devices. Transpower came to the MISG with a Simulink [4] model of the power system which accounted for multiple types of generators and the different levels of control within the power grid. Figure 1 gives the top level of this model and Figure 2 the next level for one of the generator types used in the simulation.

This model was clearly too complex for any analytical analysis of its properties and was also considered too complex for the initial investigation of alternative strategies for the grid friendly devices. Although the model had many parts the two dominant parts of the model were the inertia in the generators and a feedback control system that maintains the grid frequency.

The next section takes these two components and develops a simplified model of the power grid with similar response characteristics to the actual grid. It should be noted that the response of the grid varies depending on the actual load and generators active at the time. For this reason we are interested in what can be deduced about the general behaviour of the grid, rather than an exact simulation of the behaviour of one particular configuration of generators and load.

4 Simplified model of power grid

The electrical power grid is such that the power generated must match the power consumed. The frequency of the alternating current power is determined by the rotation speed of the generators and the power input to the generators is adjusted to maintain a nominal frequency of 50 Hertz. A sudden change in the balance of the power generated caused by, for instance, a loss of some generation capacity causes a voltage drop, typically seen as the dimming of lights, and requires extra power from the remaining generators. The generators initially supply some of the required additional power from the inertial energy stored in their rotational motion. Extracting this energy reduces the speed of the generators and hence the frequency seen on the power grid.

A feedback control is used to adjust the power going to one of the generators to bring the frequency back to the standard 50 Hertz. The time taken to adjust the power to the generators depends on the inertia in the generators and the properties of the feedback control system. Typically a disturbance results in a dip in frequency lasting a few tens of seconds. Too large a disturbance on the grid will invoke protection circuits that shut down power usage and generation, causing major blackouts.

The effective performance of the power grid can be described by (1), which is a balance of the following terms: an inertial term for the power available from the inertia of the generators as frequency changes ($mdf(t)/dt$), the net variation of power with frequency $k_f(f_0 - f(t))$ where f_0 is the normal frequency which is 50 Hertz in New Zealand, the power supplied by the controlled generator $p_c(t)$, the power supplied by other generators $p_g(t)$, and the power required by users $p_u(t)$

$$m\frac{df(t)}{dt} = k_f(f_0 - f(t)) + p_c(t) + p_g(t) - p_u(t) \quad . \tag{1}$$

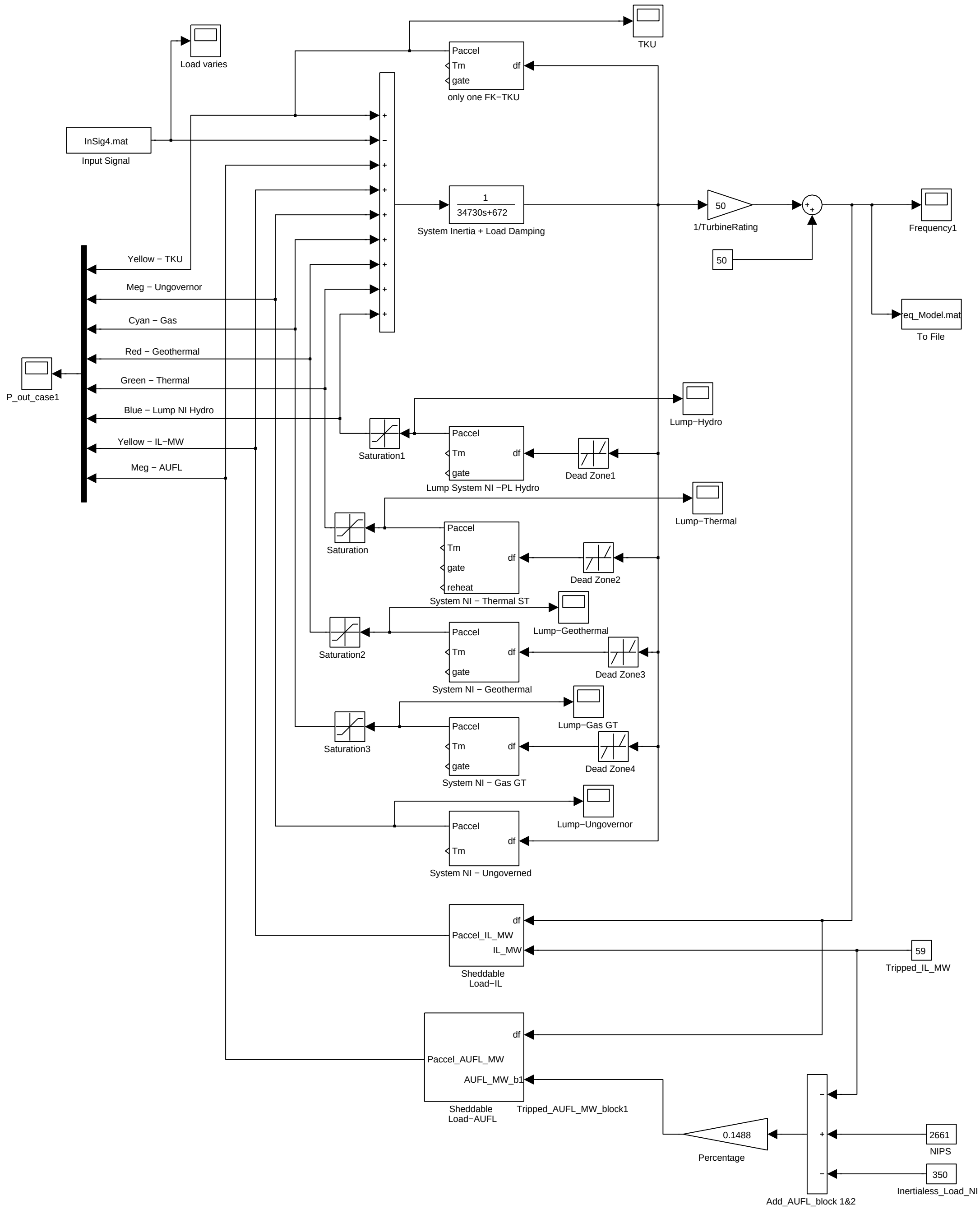

Figure 1: Top level of Transpower Simulink grid model.

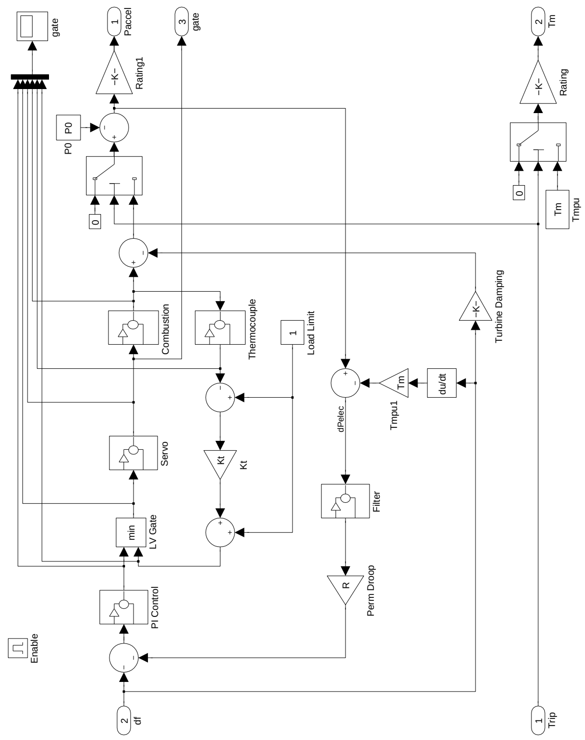

Figure 2: Detail of a generator model in Transpower Simulink model.

The feedback controller is a standard construction, known as proportional plus integral or PI, that consists of two terms, the first proportional to the error in the frequency $(f_0 - f(t))$, and the second proportional to the integral of the error. The two proportionality constants (k_p & k_i) are used to match the controller to the requirements of the process being controlled. The output of this controller sets the power going to the controlled generator. Note that if the frequency is low the controller needs to increase the power to the generator. So the equations describing the controller are

$$p_c(t) = k_p(f_0 - f(t)) + k_i x(t) \quad , \tag{2}$$

where the integral component $x(t)$ is generated by

$$\frac{dx(t)}{dt} = f_0 - f(t) \quad . \tag{3}$$

It is assumed that the control acts promptly in altering the power output of the controlled generator. The controlled generator is chosen for its ability to respond quickly to changes requested by the feedback controller.

For the evaluation of the response of a controlled system it is best to start with the system in steady state and then look at the effect of a disturbance. Thus if an initial (at $t = 0$ for convenience) steady state operation is assumed

$$f(0) = f_0, \quad x(0) = p_c(0)/k_i, \quad p_c(0) = p_u(0) - p_g(0) \ . \tag{4}$$

Equations (1), (2) and (3) depend on four parameters (m, k_f, k_p, k_i) which vary according to the grid being simulated, the generators currently being used, and the amount of power being consumed.

For analysis it is convenient to look at variation about the steady state and then reduce to a minimum the number of parameters involved. Selecting scale factors for t and x that simplify the equations gives the following transformed variables with steady state values of zero

$$T = t\sqrt{k_i/m}, \quad F(T) = f(t) - f_0, \quad X(T) = -(x(t) - x(0))\sqrt{k_i/m}. \tag{5}$$

This scaling allows the parameters to be combined into a single parameter

$$K = \frac{(k_f + k_p)}{m\sqrt{k_i/m}} \ , \tag{6}$$

and finally the power difference can be scaled to a corresponding change in frequency

$$P_d(T) = P_d(t\sqrt{k_i/m}) = \frac{p_u(t) - p_g(t) - (p_u(0) - p_g(0))}{(k_f + k_p)} \ . \tag{7}$$

It should be noted that the scaling of power to Hertz is system dependent, thus working with the scaled values has the advantage of being independent of the particular grid parameters. The equations could be made non dimensional by dividing by the standard frequency, however as this gives no further advantage, it is more convenient to retain the frequency dimension for $F(T)$.

Hence starting with the frequency and controller equations (1), (2) & (3), eliminating $p_c(t)$, using equations (5) to eliminate t, $f(t)$ and $x(t)$, substituting the values of K and $P_d(T)$ (e.g. by eliminating k_f & $p_u(t)$), and applying the initial steady state conditions to eliminate $x(0)$ and $p_c(0)$ gives

$$\frac{dF(T)}{dT} = -K\{F(T) + P_d(T)\} - X(T), \quad \text{and} \quad \frac{dX(T)}{dT} = F(T). \tag{8}$$

This shows that other than a scaling of the time axis, there is just one parameter that controls the shape of the response to a disturbance from $P_d(T)$. The initial values $F(0)$, $X(0)$ and $P(0)$ are zero for the initial steady state.

Figure 3 shows the range of responses to a step change in $P_d(T)$ for this system as the value of K is changed. The amplitude of the response $F(T)$ depends on the amplitude of the disturbance. Depending on the value of K the response changes from a damped oscillation to an exponential decay.

The speed of response in real time t, for a given K value can be increased by increasing k_i and k_p in the manner that maintains K constant. The above equations appear to indicate the response speed can be increased indefinitely, however physical limits apply to the range and change rate of the controlled generator. These limit the speed at which the system can return to steady behaviour. In addition the assumed model of the grid response has limitations in that it is only modelling the slower responses of the grid, has neglected the controls within

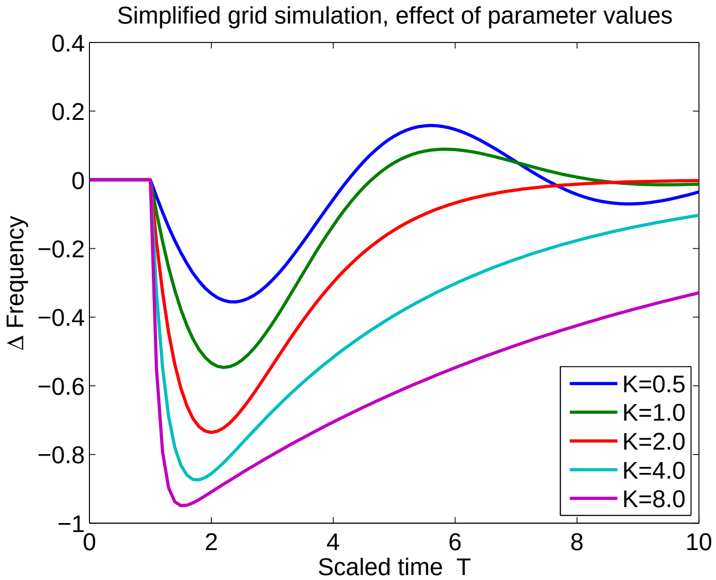

Figure 3: Possible responses of the grid to a step change in power generation, using scaled time. A step change of -1 in frequency, corresponding to a loss of generation capacity, was applied at time 1.

the individual generators and the actual implementation of the controller which could be a discrete approximation to the controller equations used above.

The controller output is $p_c(t)$ which can be obtained from the scaled values as

$$p_c(t) = -k_p F(T) - k_i \left(X(T)/\sqrt{k_i/m} - x(0) \right) \tag{9}$$

and this can be scaled similar to the other power values to give the scaled controller output

$$C(T) = -\frac{k_p F(T) + k_i \left(X(T)/\sqrt{k_i/m} - x(0) \right)}{k_p + k_i} \quad . \tag{10}$$

The range of this variable is determined by the power range of the controlled generator.

Comparing the responses in Figure 3 with an actual response will allow the time scale factor $\sqrt{k_i/m}$ and the value of K to be determined. By combining these, other ratios of the original parameters can be obtained. The values of k_p and k_i can be obtained from the physical controller implementation together with its input and output scale factors. We then have sufficient information to determine the other two original parameters, namely m and k_f.

Inspection of an actual grid response to the loss of a generator supplied by Transpower figure 4, shows that this simple model with a value for K of about 1.5 gives a response similar to that seen in this example. The responses of the power grid to a disturbance vary to some extent depending on the load and generators in use at the time of the disturbance.

The simple model provided two significant advantages: it enabled some analytical analysis of the system behaviour and it provided a convenient system on which to test the effect of various grid friendly devices.

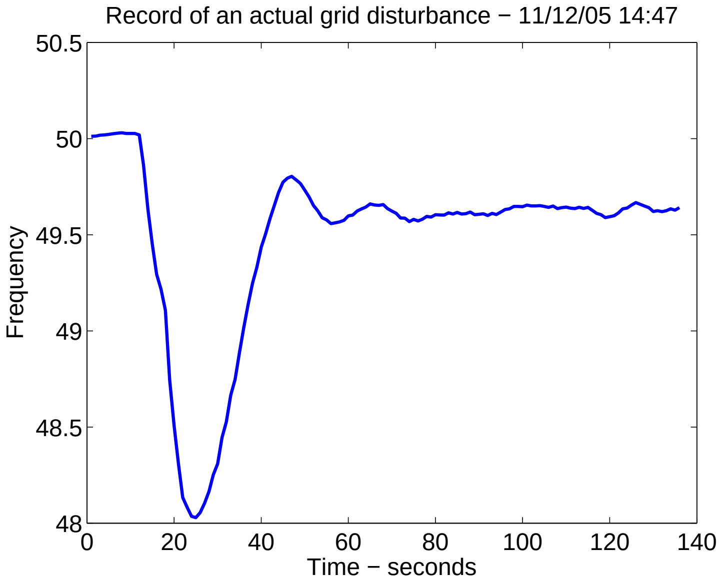

Figure 4: Recorded effect of a step change in power generation on the Transpower grid.

The eigenvalues of the system given by equations (8) are

$$-K/2 \pm \sqrt{(K/2)^2 - 1} \tag{11}$$

which always have a negative real part and hence this model of the control system is stable for all positive values of m, k_f, k_p and k_i. For small values of K the solution is a decaying oscillation, and for values of K greater than 2 the solution consists of two decaying exponentials.

Note however the model is an approximation that neglects certain aspects of the physical system that would add small higher order terms to the differential equations and possibly change the controller differential equations to difference equations. These could result in the control becoming unstable at high values of the gains k_p and k_i.

5 Inclusion of grid friendly devices

The goal of a grid friendly device is to make a change in the power consumption based on the frequency it sees. This appears in the power equation (1) as part of the power consumption required by the users $p_u(t)$. The power used by the grid friendly devices is determined by their current need for power and by the frequency they see on the grid. In the following we will assume that the need for power is a constant, and the actual amount of power used is reduced based on some algorithm that uses measurements of the frequency.

The effect of the grid friendly devices is conveniently included by removing their effect from the scaled net power $P_d(T)$ and defining the effect of the grid friendly devices as $G(F)$ which is scaled in frequency similarly to $P_d(T)$. Define $P(T)$ as

$$P(T) = P_d(T) - G(F(T)) \tag{12}$$

so that equation (8) becomes

$$\frac{dF(T)}{dT} = -K\{F(T) + P(T) + G(F(T))\} - X(T) \quad . \tag{13}$$

Here $G(F)$ by appropriate choice of the base power level in (7) can be assumed to be zero for normal operation and negative when the frequency drops and the device is not consuming power. This allows steady state values of zero which will be useful for an analysis using stability conditions.

The grid friendly devices are represented here by the function $G(F)$, which is not necessarily a simple function of F as it can also use a memory of past behaviour and possibly incorporate time delays. A major aim of this MISG project was to determine the effect of different algorithms for the behaviour of the grid friendly devices on the grid. From this recommendations for the best designs for the grid friendly devices can be made.

The Simulink diagram for the grid with the inclusion of a grid friendly device is given in Figure 5. This can be compared with Figures 1 and 2 to see how the essential behaviour of the grid is much simpler than a full simulation.

5.1 Stability analysis for grid friendly devices

Equations (8) and (13) describe the behaviour of the combined power grid and grid friendly devices. For a steady state there should be no disturbances entering the grid, which means $P(T)$ is constant. It is then possible to choose $x(0)$ in equation (5) so that $P(T)$ trends to zero after an initial disturbance, such as the loss of a generator. Now consider the energy like term

$$E(T) = F(T)^2 + X(T)^2 \quad . \tag{14}$$

It is clear that if $E(T)$ goes to and stays at zero, $F(T)$ and $X(T)$ have reached steady states of zero. Differentiating this with respect to T, and substituting from the differential equations (8) and (13) gives

$$\begin{aligned} \frac{dE(T)}{dT} &= 2F(T)\frac{dF(T)}{dT} + 2X(T)\frac{dX(T)}{dT} \\ &= 2F(T)\{-K\{F(T) + G(F(T))\} - X(T)\} + 2X(T)F(T) \\ &= -2KF(T)^2 - 2KF(T)G(F(T)). \end{aligned} \tag{15}$$

With $G(F(T))$ zero, (13) and (8) are linear, and hence $E(T)$ must decrease to zero, as its derivative is negative or zero and can only remain zero when both $F(T)$ and $X(T)$ are zero.

As $G(F(T))$ is zero when the grid friendly devices are switched on and negative when they are switched off, the term $F(T)G(F(T))$ can be guaranteed to be positive, provided the grid friendly devices are on (i.e. consume power, $G(F(T)) = 0$) when the frequency is equal to or above the normal frequency ($F(T) \geq 0$). Under this condition the effect of $G(F(T))$ is to make the derivative more negative and hence speed the decrease of $E(T)$ to its stable zero value.

Thus provided the grid friendly devices are only switched off when the frequency is lower than the normal frequency the grid response is stable and further the effect of the grid friendly devices is to improve the stability by creating a faster return to normal conditions.

It should also be noted that large values of K (6) will cause E to reach zero faster and hence a quicker return to stable conditions. This is also seen in the eigenvalues given in (11).

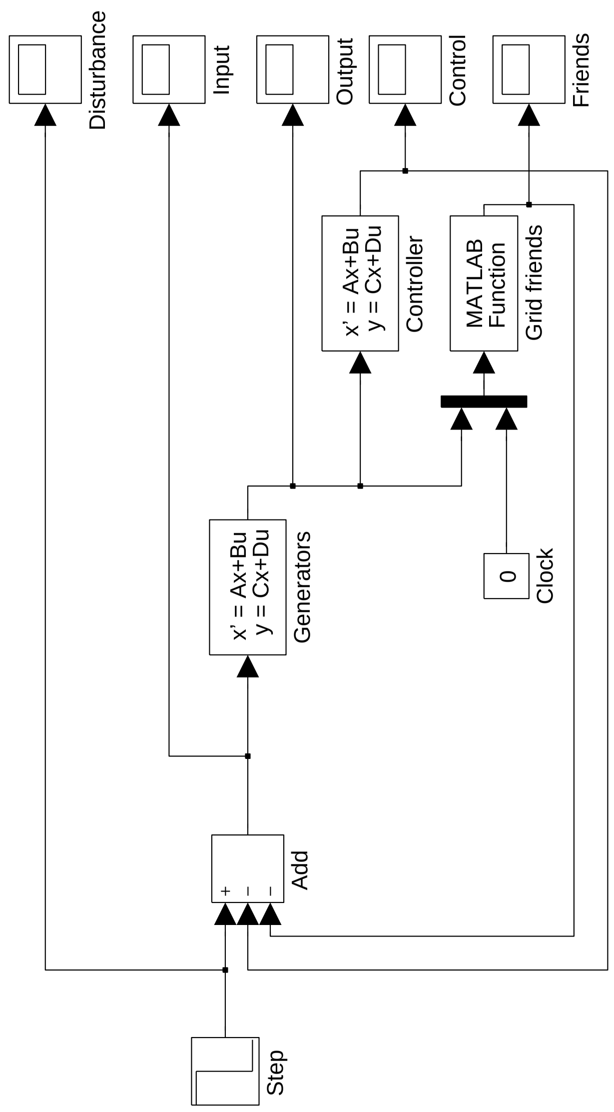

Figure 5: Simulink model of simplified grid equations with grid friendly devices.

This analysis has looked at the stability in terms of the two state variables $F(T)$ and $X(T)$. However it is also desirable that the right hand side of equation (13) remains smooth. In particular repeated switching of large loads on and off is to be avoided. Hence an examination of performance needs to look at both the frequency and the input to the frequency equation.

5.2 An additional stability result

For the case where the frequency response is non oscillatory ($K \geq 2$) and the grid friendly devices act to turn off power progressively at frequencies less than f_0 and also turn the devices back on progressively as the frequency returns to f_0 (but not at a value lower than the turn off frequency) it is possible to derive an additional stability result.

It can be shown that in this case the load changes made by the grid friendly devices maintain a non oscillatory response. Over the range where the grid friendly devices operate proportional to frequency with a positive slope (i.e. less power is used when the grid frequency is lower), their effect is to change the response to that obtained with a larger value of K, which corresponds to an increase in the amount of damping. The derivation of this result is given in the appendix.

6 Simulation of grid friendly devices

The grid friendly devices are actually multiple independent units. When they all switch at the same frequency they can be considered as a single device. Similarly when they switch over a range of frequencies the continuous approximation of a single device can be used. When the devices use a more complex algorithm that involves a memory of past behaviour in the individual devices, it becomes difficult to express this as a single device that integrates the behaviour of the individual devices. For this reason the simulations below have been done using a simulation of a sufficient number (500) of individual grid friendly devices each implementing the algorithm under test.

The following subsections look at different possible implementations of the grid friendly devices. In the formulas where it has been possible to describe the units as a single device this has been done using Q as the total frequency change, and where the formula for a single unit needs to be used a frequency change of q for each unit is used. For comparison, the behaviour of the grid for the same disturbance without any grid friendly devices is given in Figure 6. Here a loss of power disturbance equivalent to a drop in frequency of 10 units occurs at time 1. This graph and the following graphs are generated using (8) & (13), and show the change in frequency (Frequency, equation (5)), the scaled output to the controlled generator (Controlled, equation (10)), and the scaled output of the grid friendly devices (Grid Friendly, introduced in equation (12)).

The controller response has been chosen for a quick recovery to the base frequency with almost no overshoot, and a minimal overshoot in the controlled power. A faster return to the normal frequency can be obtained with a larger overshoot in the controller output, however this is undesirable and may not be feasible. For this reason it is always necessary to check that the controller output is acceptable, as well as checking the response of the controlled variable (frequency in this case). The parameters used in the following simulations are: $k_f/m = 1$, $k_p/m = 1$, and $k_i/m = 1.5$ which give $K = 1.63$.

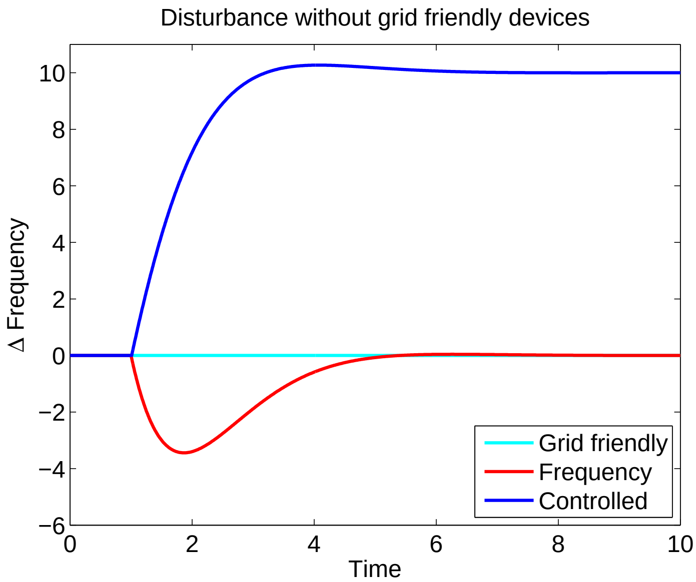

Figure 6: Response of the simulated grid to a disturbance without any grid friendly devices.

6.1 Simple grid friendly devices

The simplest case for the grid friendly function is switching off if the frequency is below a given value ($a < 0$) that is

$$G(F) = \begin{cases} 0, & F > a, \\ -Q, & F \leq a < 0. \end{cases} \tag{16}$$

Figure 7 shows a typical response of this device. Compared with Figure 6 the grid friendly devices have reduced the size of the disturbance in the frequency, but as the grid recovers the grid friendly devices switch on and off rapidly as the switch frequency is crossed, and this holds the frequency at the switching frequency until adjustment from the grid friendly devices is no longer needed. During recovery once the switching frequency is reached the oscillation in the devices delays the recovery of the power grid to the normal frequency. In the case where the grid friendly devices control more power than the disturbance the oscillation starts as soon as the switching frequency is reached.

While this case improves the frequency response by reducing the size of the frequency overshoot, the rapid switching of the devices at the switching frequency delays recovery and is quite undesirable.

6.2 Addition of dead band

To avoid the rapid switching of the grid friendly devices a dead band can be introduced. For this case it is necessary to introduce a memory for the state of the device output when the frequency is in the dead band region $-a < F < -b$ ($a > b > 0$), as in this region there are two possible outputs (0 or $-Q$). The algorithm uses an internal memory Z to remember the state of the output when F is in the dead band and can be implemented as

$$G(F) = Z$$

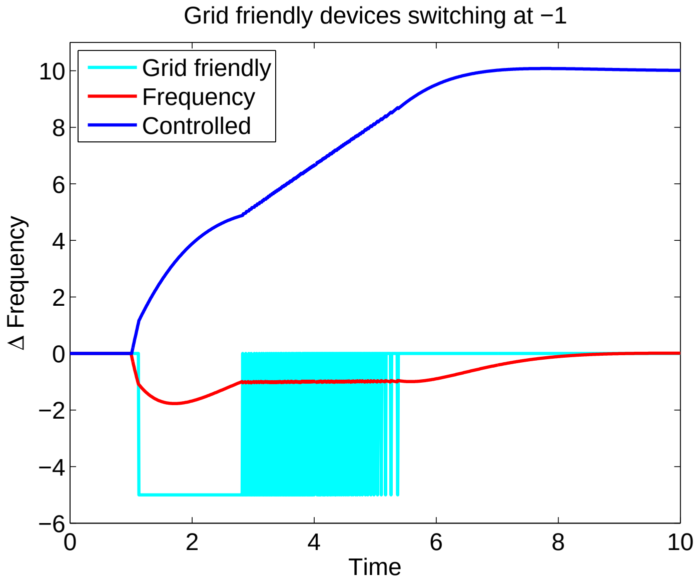

Figure 7: Response from a simple grid friendly device switching at a given frequency.

$$\begin{aligned}&\text{if } (F > -b) \quad G(F) = 0\\ &\text{if } (F < -a) \quad G(F) = -Q\\ &Z = G(F) \quad .\end{aligned} \tag{17}$$

The value of Z needs to be initialised to zero before this algorithm is run.

As seen in Figure 8 there is again an improved frequency response. However if the grid friendly devices control enough power there will be oscillation between the two switching frequencies. A larger dead band reduces the possibility of oscillation and reduces the frequency of oscillations that do occur. The higher value of the switch on frequency allows a recovery to closer to the normal frequency, before the switching back on of the grid friendly devices acts to delay the recovery to the normal frequency. The repeated switching of the grid friendly devices is also seen in the controlled power as an undesirable saw tooth as the controller output tries to adjust to the oscillating power usage.

6.3 Addition of a delay before switching back on

This case is similar to that used in the grid friendly demonstration trial [1] and investigated in the paper by Lu and Hammerstrom [2]. Here the grid friendly devices do not switch back on until a given time after the frequency has recovered to a value above the switch off level. To implement this access to the time (T) or a count down timer is needed in the grid friendly devices. Here a is the switching level below the normal frequency and c is the time delay

$$\begin{aligned}&G(F) = Z\\ &\text{if } (F < -a) \quad G(F) = -Q; \quad t_0 = T\\ &\text{if } (F > -a \;\; \& \;\; T - t_0 > c) \quad G(F) = 0\\ &Z = G(F) \quad .\end{aligned} \tag{18}$$

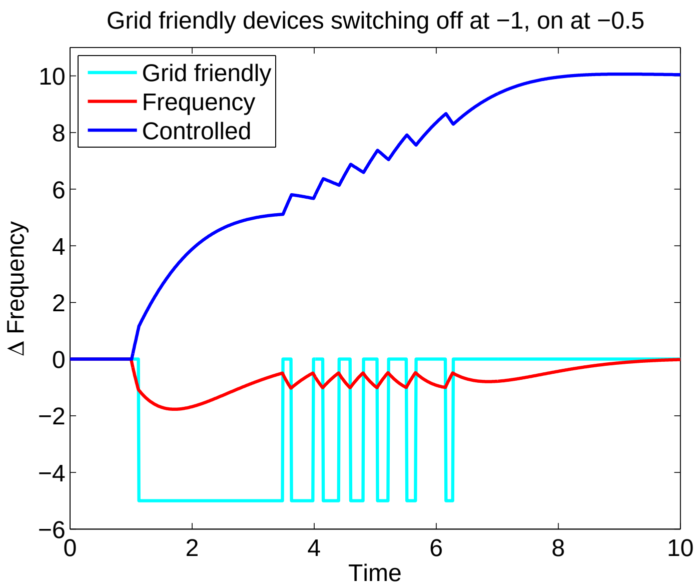

Figure 8: Response from a simple grid friendly devices switching at a given frequency.

This needs to have initial values of $Z = 0$ and $t_0 = T$ set before the algorithm is applied:

In this case the switching on of the devices occurs without consideration of the effect on the grid. When the devices control sufficient power the effect of them switching on may create a disturbance sufficient to drive the frequency low enough to cause the devices to switch off again, starting an oscillation. Figure 9 shows how this type of grid friendly device can give undesirable oscillations. For smaller amounts of grid friendly power there may not be any oscillation, or there could be a few cycles before the grid returns to the steady condition.

6.4 Spreading the switching frequency

Here the switching frequency is spread over a range by each device switching at a different frequency within a predetermined range. The actual frequency used by each device could be set during manufacture, or better, determined by random selections during operation which would make the average performance of each device the same. In this case it is possible to describe the integrated behaviour of the grid friendly devices as $(a > b > 0)$

$$G(F) = \begin{cases} 0 & F > -b < 0 \\ \text{-((b+F)/(b-a)) Q} & -b \geq F \geq -a \\ \text{-Q} & -a > F. \end{cases} \tag{19}$$

This has the advantage of giving a proportional response that is able to adjust to the size of the disturbance. In particular it avoids the possibility of too large a drop in load and the frequency going above the desired 50 Hertz value. Figure 10 shows the effect of this algorithm. With the switching range chosen suitably the reduction in the frequency drop is similar to switching at a given value, but the return to using power by the grid friendly devices is now smooth. The return of the grid friendly devices does slow the recovery to the normal frequency. However the controller output is now back to an acceptable smooth behaviour.

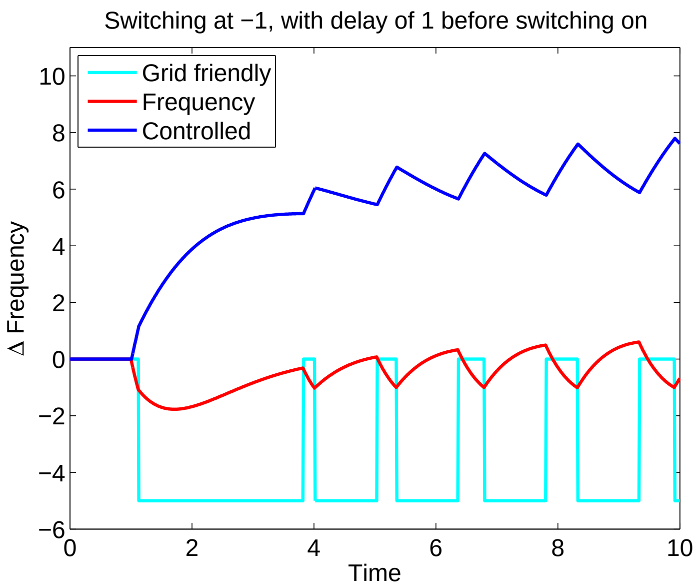

Figure 9: Response from grid friendly devices with a time delay before switching back on.

6.5 Spread switching frequency with dead band

The above sections describe the grid friendly devices as a single unit that combines the effect of the multiple devices. This is difficult to implement for this case and instead a description of one device is given and this is used multiple times in the simulation. To obtain a spread of frequencies, the switch frequencies for the individual devices are determined within the range allowed using random numbers. The frequency range used to switch the device off is from a_{off} to b_{off} ($a_{off} < b_{off} < 0$) and for switching the device back on a_{on} to b_{on} ($a_{on} < b_{on} < 0$ and $a_{off} < a_{on}$, $b_{off} < b_{on}$). The algorithm for each device is

$$
\begin{aligned}
& G(F) = z \\
& \text{if } (F > b_{on} \ \ \& \ \ z < 0) \\
& \qquad r = \text{random}(0,1) \\
& \qquad s_{on} = a_{on} + r(b_{on} - a_{on}) \\
& \qquad s_{off} = a_{off} + r(b_{off} - a_{off}) \\
& \qquad G(F) = 0 \\
& \text{if } (F < a_{off}) \ G(F) = -q \\
& z = G(F) \quad .
\end{aligned}
\tag{20}
$$

Similar to the case in Section 6.2 initialisation of the internal variables is needed. This is

$$
\begin{aligned}
& z = 0 \\
& r = \text{random}(0,1) \\
& s_{on} = a_{on} + r(b_{on} - a_{on}) \\
& s_{off} = a_{off} + r(b_{off} - a_{off}) \quad .
\end{aligned}
\tag{21}
$$

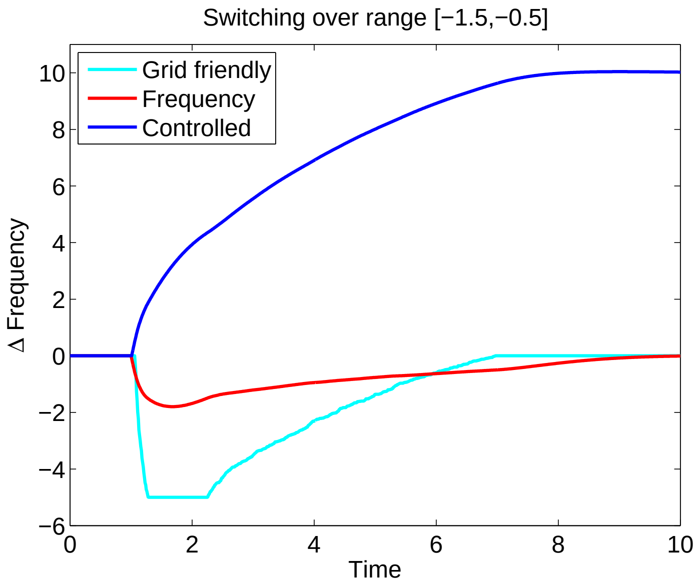

Figure 10: Response from a grid friendly device switching over a range of frequencies.

Note here the change in power is q for one device and the total possible change is the number of devices times q. The total effect of these grid friendly devices (Figure 11) is obtained by adding up the effects of each of the devices. Typically the simulations use 500 devices, which is sufficient to give a close estimate of the effect of many devices.

Similar to the previous case there is an offset from the target frequency as the devices turn back on. However the dead band allows the offset to be moved closer to the normal frequency.

6.6 Addition of a time delay before switch on

Here a range of switch off frequencies are used, but the condition for switching back on includes a random delay after the frequency has recovered beyond the switch on frequency, which is also given a range of values. The random time delay gives the grid frequency more time to recover before the additional load is placed on the grid and ensures there is a progressive addition of load to the grid. To implement the time delay there needs to be access to a clock (T the value of which increases with time), as in the following

$$
\begin{aligned}
&G(F) = z \\
&\text{if } (F > b_{on} \;\; \& \;\; z < 0 \;\; \& \;\; T - t_0 > c) \\
&\quad\; r = \text{random}(0,1) \\
&\quad\; s_{on} = a_{on} + r(b_{on} - a_{on}) \\
&\quad\; s_{off} = a_{off} + r(b_{off} - a_{off}) \\
&\quad\; c = c_{min} + r(c_{max} - c_{min}) \\
&\quad\; G(F) = 0 \\
&\text{if } (F < a_{off}) \; G(F) = -q
\end{aligned}
$$

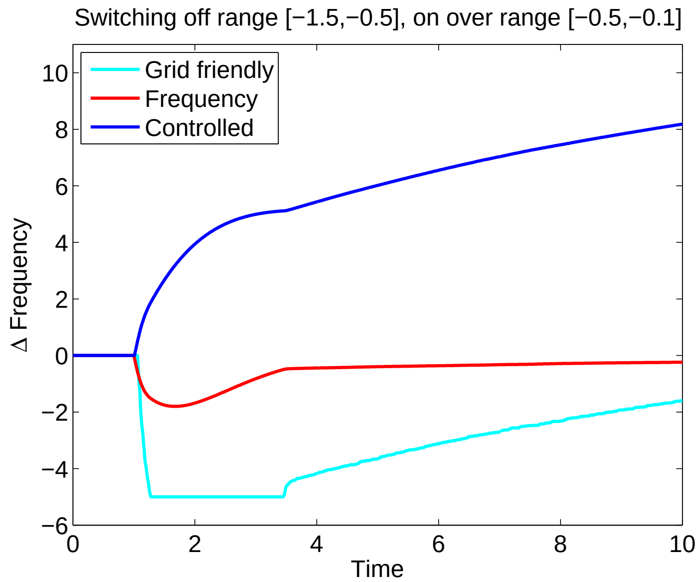

Figure 11: Response from simple grid friendly devices switching over a range of frequencies and using a dead band.

$$\begin{aligned} &\text{if } (F < a_{on}) \; t_0 = T \\ &z = G(F) \quad . \end{aligned} \tag{22}$$

Again an initialisation of the internal variables is needed. This is

$$\begin{aligned} &z = 0 \\ &r = \text{random}(0,1) \\ &t_0 = T \\ &s_{on} = a_{on} + r(b_{on} - a_{on}) \\ &s_{off} = a_{off} + r(b_{off} - a_{off}) \\ &c = c_{min} + r(c_{max} - c_{min}) \quad . \end{aligned} \tag{23}$$

This gives the best performance (Figure 12) in that the switching off of the grid friendly devices occurs in proportion to the size of the disturbance, and switching back on is delayed until the frequency has largely recovered. The distribution of time delays brings in the devices slowly after the frequency has recovered sufficiently. Should the extra load from switching some devices back on be sufficient to drop the frequency, others will delay switching back on until the frequency has recovered sufficiently.

7 Comments on simplified model

There are three main approximations made in the simplified model:

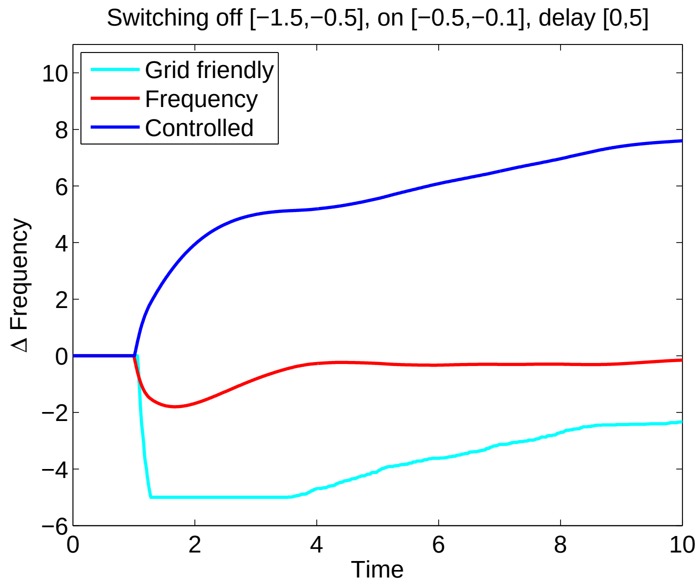

Figure 12: Response from grid friendly devices switching over a range of frequencies and using both a dead band and a delay.

1. The process model for f actually contains additional terms that become significant when the process is required to respond faster than the time scales for which the model was developed.

2. The control is implemented by equipment that is not able to respond at very high speeds. Typically it is implemented using discrete time steps and may be rate limited.

3. The grid friendly devices do not implement a continuous switching of power, but switches network load in discrete steps.

If the feedback gains k_p, k_i or $G(F)$ become too great these approximations become important and the grid system could become unstable. The levels at which this occurs need to be tested on the more complex model.

8 Conclusions

The simplified model has proved very useful for analysis of stability conditions, and for testing the effect of various designs for the grid friendly devices.

The grid friendly devices have a clear ability to assist in making the grid more tolerant to disturbances by decreasing load when there is a drop in generation capacity.

Provided the power being switched by the grid friendly devices is not too large, compared with the disturbance that initiates switching, it makes little difference whether they all switch off at the same frequency or switch off over a range of frequencies. If the grid friendly devices control a larger amount of power switching over a range of frequencies allows the amount of load reduced to be proportionate to the power loss. This then avoids too large a correction that causes the grid frequency to rise above the normal value.

The effect of the grid friendly devices when they switch back on after a disturbance has the potential to cause problems on the grid. If they all switch on at the same frequency or time they create another disturbance and drop in frequency, which may well be sufficient to cause the devices to switch off again, possibly multiple times. To avoid this the switching back on of the grid friendly devices needs to be spread out, either over a frequency, over time or both. Spreading out over frequency even when offset from the switch on frequencies has the effect of delaying recovery to the normal frequency. So the best option is switching back on after a variable delay, provided the frequency has recovered sufficiently, with the recovery frequency value being spread over a range of frequencies.

Acknowledgements

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Appendix: Additional stability result

In this appendix an alternative stability result is derived for the case where the control response is not oscillatory, that is for critically or over damped responses. It is found in this case that a wide range of grid friendly device designs, increase the amount of damping on the power grid.

Consider equations (1), (2) & (3), assume p_g constant, and $p_u(t) = a + bf(t)$ with $a \geq 0$ & $b \geq 0$, for some range of $f(t) < f_0$ which corresponds to the grid friendly devices reducing power usage as the frequency decreases, and increasing power usage when the frequency rises. It is assumed that power usage function corresponding to the combined action of the grid friendly devices, can consist of piecewise sections. This provides a method of specifying a class of grid friendly devices that includes the range of typical implementations.

Differentiate equation (1), replace df/dt by y, substitute for p_c from (2), and substitute for dx/dt from (3) to get

$$\frac{df(t)}{dt} = y(t), \quad \frac{dy(t)}{dt} = -\frac{k_f + k_p + b}{m} y - \frac{k_i}{m}(f(t) - f_0). \tag{24}$$

The condition for these equations to be non oscillatory without the effect of the grid friendly devices (i.e. $b = 0$) is

$$(k_f + k_p)^2 \quad \geq \quad 4mk_i \quad . \tag{25}$$

When $b > 0$ the left hand term in (25) becomes $(k_f + k_p + b)^2$ and the inequality still holds.

Figure 13 shows a solution to (24) and defines three regions. For a loss of generation capacity $f(t)$ is initially less than f_0. Now consider the (f, y) plane:

In the quadrant $f < f_0$ & $y \leq 0$ (region A): $dy/dt > 0$ & $df/dt \leq 0$ so the solution to (24) always increases in y and exits the quadrant through $y = 0$.

In the quadrant $f \leq f_0$ & $y > 0$ (regions B & C): $df/dt \geq 0$ so that the solution of (24) always travels towards the y axis.

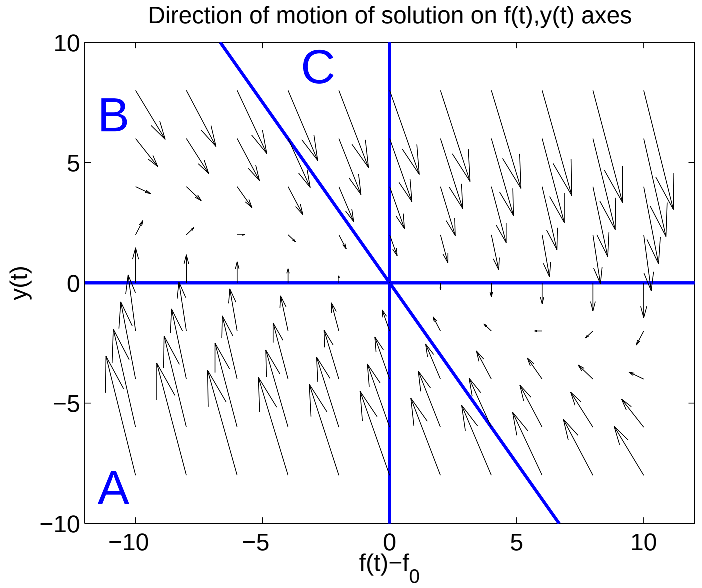

Figure 13: Solution to equations (24) shown as vectors giving magnitude and direction of the solution at various points in the $f(t), y(t)$ plane. The three regions marked are those referred to in the text. Parameters used for this plot are $(k_f + k_p)/m = 3$, $b = 0$ and $k_i/m = 1$. The sloping line is $3(f(t) - f_0) + y(t) = 0$.

On the line

$$(k_f + k_p)(f - f_0) + 2my = 0 \tag{26}$$

the y derivative from (24) becomes

$$\frac{dy}{dt} = -\frac{k_f + k_p + b}{m}\left(-\frac{(k_f + k_p)(f(t) - f_0)}{2m}\right) - \frac{k_i}{m}(f(t) - f_0)\ . \tag{27}$$

Using the inequality (25) this becomes

$$\frac{dy}{dt} \leq -\left(\frac{(k_f + k_p + b)(k_f + k_p)}{2m^2}(f(t) - f_0) - \frac{(k_f + k_p)^2}{4m^2}\right)(f(t) - f_0) \tag{28}$$

and this after simplification is

$$\frac{dy}{dt} \leq \frac{(k_f + k_p)(k_f + k_p + 2b)}{4m^2}(f(t) - f_0)\quad . \tag{29}$$

Thus dy/dt is negative on the line (26) for $f < f_0$. The f derivative on the line (26) becomes

$$\frac{df}{dt} = -\frac{(k_f + k_p)}{2m}(f(t) - f_0) \tag{30}$$

which is positive for $f < f_0$.

From (29) & (30) on the line (26) in the (f, y) plane, the slope of the solutions to (24) is

$$\frac{dy/dt}{df/dt} \leq -\frac{(k_f + k_p + 2b)}{2m} \leq -\frac{(k_f + k_p)}{2m}, \tag{31}$$

where the last expression is the slope of the line (26). Hence the solution of (24) crosses the line in the downward direction when $f < f_0$ (or in the limiting case moves along the line).

Hence once the solution to (24) is in the region $f \leq f_0$ between the lines (26) and $y = 0$ (region B, Figure 13), it moves in the direction of increasing f until it reaches the stable point $f = f_0$, $y = 0$. Solutions that start with $f \leq f_0$ but $y < 0$ (region A) enter the region between lines (26) and $y = 0$ (region B) across the line $y = 0$ and thus then go to $f = f_0$ & $y = 0$.

When the grid friendly devices are implemented as joined piecewise sections with $b \geq 0$ for each section, the solution of (24) is continuous and each section of the solution that starts in the region $f < f_0$ & $y < (k_f + k_p)(f - f_0) + 2my$ (regions A & B, Figure 13) remains in this region. Hence the solution once in this region remains in the region, and heads towards the stable point $f = f_0$ & $y = 0$.

The effect of grid friendly devices implemented in this manner is to increase the amount of damping in the solution of equations (24).

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