A Theory of Chronic Loss, Suffering and Alcoholism

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AND ALCOHOLISM

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Abstract: This paper focuses on the consumption of alcohol to numb the suffering associated with failure. While drinking reduces the individual’s current level of suffering, it leads to future failures and potentially greater suffering. The basic model shows that the stationary status of an alcoholic is improved by the difference between his rate of time preference and the rate of return on his status and that this improvement is amplified by the ratio of the instantaneous suffering-relief effect to the status-eroding effect of alcohol. The extended model shows that society’s reaction to alcoholism may lead to permanent cyclical alcohol consumption.

Key words: Alcohol consumption; Limit cycles

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A THEORY OF CHRONIC LOSS, SUFFERING AND ALCOHOLISM

I. Introduction

Attempts to provide an economic rationale to a persistent and significant consumption of alcohol are based on the notion of rational addiction. Stigler and Becker [1977], Becker and Murphy [1988] and many other psycho and socio-economists constructed models of rational addiction in which forward-looking utility maximization is used to explain observed addictive behavior. These rational addiction models propose that rational planning stemming from lifetime-utility maximization and addiction are not incompatible, that when dealing with addictive goods unstable steady states are a common characteristic, and that these unstable steady states imply that small deviations in current consumption can lead to large cumulative changes. Models of rational addiction were subjected to empirical tests and applied to the analysis of the consumption of alcohol by Waters and Sloan (1995), Grossman, Chaloupka and Sirtalan (1998) and others. As in the case of any other good, it is postulated in these studies that the consumption of alcohol increases the individual instantaneous utility. However, no reference to specific benefits from this consumption is provided.

A Homo Sapience’s motivation and behavior might diverge from constrained-utility maximization (Thaler, 2000, Rabin and Thaler, 2001) and a higher income might not necessarily lead to a greater happiness. According to contents theories of motivation (Murray, 1938; Maslow, 1943, 1945; Herzberg, 1966; Alderfer, 1972), a higher income
may “buy” more happiness for the poor but it might not for the rich. Indeed, suffering, depression and alcoholism are widespread, interrelated problems in affluent societies (e.g., Sitharthan et al., 2001).

This paper diverges from the utility-maximizing paradigm to provide a conceptual framework for analyzing the phenomenon of suffering and the persistent consumption of alcohol. The paper focuses on the consumption of alcohol to numb the suffering accompanying frequent and, possibly, interrelated losses and failures (e.g., in the areas of career, finance, relationships, marriage, family). Suffering is used as a generic term representing the effects of loss and failure on the individual: deprivation, humiliation, degradation, anger, impotence, sadness, depression and grief. Similarly, alcohol is used as a generic term representing alcoholic beverages and/or drugs and as an example of a substance that eases instantaneous suffering.

The proposed analysis is focused on chronic losers who are inclined to consume alcohol, and describes their optimal consumption of this substance. It stresses the inter-temporal trade off associated with the consumption of alcohol. While drinking alleviates the individual current suffering, it erodes the individual’s status (through loss of concentration, self control, health and trustworthy reputation) and thereby leads to greater potential future suffering. In order to capture this inter-temporal trade off, the paper employs a dynamic framework which considers the chronic loser’s suffering and alcohol consumption over his lifetime. The alcohol consumption path which minimizes the chronic loser’s lifetime suffering stemming from the distance between his desired status and actual status is considered to be rational.

One exception is Brito and Strain (1996) that use the results from the biomedical literature to construct a
In its basic version the model shows that a rational alcoholic chronic loser’s rate of time preference exceeds the rate of return on his status. It is also demonstrated that the stationary status of a rational alcoholic chronic loser is improved by the difference between his rate of time preference and the rate of return on his status and that this improvement is amplified by the ratio of the instantaneous suffering-relieving effect (i.e., \textit{the blessing}) of alcohol to the status-eroding effect (i.e., \textit{the curse}) of alcohol.

When society’s reaction to excessive alcohol consumption is taken into account, it modifies the basic model in important ways. As a consequence, a different and more complex dynamic path for alcohol consumption is generated. Society reacts to alcoholism because it creates huge costs and negative externalities. It is assumed that the role of society is to provide help and support to chronic alcoholics. The modified model is able to show that cycles in alcohol consumption arise when the rate of return on status is greater than the rate of time preference.

The paper is structured as follows. Section II introduces a set of assumptions portraying the characteristics of an alcoholic chronic loser and the effect of his alcohol consumption on his present and future level of suffering. Using this set of assumptions, section III derives the no-arbitrage rule of alcohol consumption for a lifetime suffering minimizing alcoholic chronic loser for the case where there is no public reaction to alcoholism. Section IV computes the stationary status and alcohol consumption for this case and display their asymptotic stability properties in a phase plane diagram. Section V analyzes the role of society in supporting the alcoholic fighting alcoholism and shows the dynamic model of alcohol consumption.
conditions that generate a permanent cyclical pattern in alcohol consumption. Section VI presents concluding remarks.

II. Characteristics of an Alcoholic Chronic Loser (ACL)

Let:
\( t \) = a continuous time index;
\( x(t)^* \) = the individual’s desired status at \( t \) - a combined index \((x^* \in \mathbb{R}_+)\) of the individual’s desired positions with regard to health, wealth, career, relationship, family, self expression, spiritual progress, etc.;
\( x(t) \) = the individual’s actual status at \( t \), \( 0 \leq x(t) \leq x^* \);
\( c(t) \) = the individual’s consumption of alcohol at \( t \);
\( s(t) \) = the individual’s suffering at \( t \);
\( g(c(t)) \) = the suffering-relief degree of alcohol consumption at \( t \);
\( r(t) \) = the individual’s rate of return on the status at \( t \);
\( \delta(t) \) = the individual’s marginal status erosion caused by drinking at \( t \) (e.g., through loss of concentration, self control, reputation and health); and
\( \rho(t) \) = the individual’s rate of time preference at \( t \).

The description of the alcoholic chronic loser and the effect of his alcohol consumption on his level of suffering and status are summarized by the following set of assumptions.

**Assumption 1 (Failure is imminent in the present location):** An ACL always fails to reach his desired status in his present physical and social environment because of one, or more, of the four following reasons.

i. **Aiming too high:** Chronic failure is inevitable when the desired status is persistently set beyond capacity.
ii. Illusive target: The individual is meritorious, but his target (or desired status) is extremely difficult to attain and persistence leads to chronic failure and misery.

iii. Eccentricity and non-conformity: Persistent private attempts to promote unconventional ideas and status lead to public alienation, ridicule, marginalization, isolation and persecution. A famous allegorical example is Miguel de Cervantes Saavedra’s self proclaimed knight - Don Quixote de la Mancha.

iv. Discrimination: Being recurrently disadvantaged on racial, ethnic, gender, appearance and handicap grounds spawns chronic feelings of deprivation.

Assumption 2 (Immobility): An ACL cannot escape failure by leaving his present physical and social environment. The history of failures, eccentric ideas and drinking render the ACL an undesired and unsuccessful immigrant (persona non grata).

Assumption 1 and 2 can be formally expressed as \( x(t)^* - x(t) > 0 \forall t \).

Assumption 3 (Independence of past and future failures): The ACL’s present level of suffering reflects only his present failure. That is, in contrast to the basic idea of rational addiction, no stock of “suffering capital” is formed by past suffering and failures. The ACL’s present level of suffering increases with his present failure’s magnitude - the distance between his current desired status and his actual status - and is eased by the numbing effect of his current alcohol consumption

\[
s(t) = g(c(t))[x(t)^* - x(t)]
\]

where the suffering-relief degree \( g \) is a convex function of \( c \) displaying \( g' < 0, \ g'' > 0, \ g(c = 0) = 1 \) and \( \lim_{c \to \infty} g(c) = 0 \).
**Assumption 4 (Persistence):** Despite his failures, an ACL does not modify (i.e., lower) his desired status. That is, \( x(t)^* = x^* \) for any \( t \). This assumption is consistent with assumption 1. It is also supported by Maslow’s (1954) theory of hierarchical needs: namely, as long as the currently principal need is ungratified it remains paramount and the individual cannot change his aspirations and effort to satisfy a higher need.

**Assumption 5 (Parameters):** To simplify the analysis, the ACL’s rate of time preference, rate of return on his actual status and marginal erosion of his status by drinking are considered to be positive scalars. They do not change with the ACL’s status and are time-invariant. That is, \( \rho(t) = \rho \), \( r(t) = r \) and \( \delta(t) = \delta \) for any \( t \).

**Assumption 6 (Status change):** The ACL’s actual status change is given by the difference between his return on his current actual status and the damage inflicted by his current consumption of alcohol:

\[
\dot{x}(t) = r x(t) - \delta c(t). \quad (2)
\]

This assumption and assumptions 1 and 2 complement one another: a failure leads to drinking (so as to relieve current suffering) and drinking damages the ACL’s status and hence leads to future failures. That is, although drinking reduces the ACL’s instantaneous level of suffering, it erodes his status and raises his potential (i.e., pre-drinking) level of suffering over the remaining lifetime.

**Assumption 7 (Non-suicidal):** Despite his permanent suffering, an ACL’s does not contemplate suicide. He prefers the alcoholic option to suicide in dealing with his present suffering. In contrast, a suicidal chronic loser is too proud to bear the degradation of
status caused by consuming alcohol and prefers ending his suffering by committing suicide\(^2\).

**III. Basic ACL Model: The Rational Choice of Alcohol Consumption in the Absence of Public Reaction**

An ACL’s alcohol consumption path is rational if it minimizes his lifetime suffering,

\[
\int_0^\infty e^{-\rho t} g(c(t))[x^*-x(t)]dt, \text{ subject to his actual status’ evolution indicated by Eq. (2)}.
\]

The Hamiltonian corresponding to this constrained minimization problem is

\[
H(t) = g(c(t))[x^*-x(t)] + \lambda(t)[rx(t) - \delta c(t)]
\]

where the costate variable \(\lambda(t)\) indicates the shadow price of the ACL’s status at \(t\). Since \(g(c)\) is convex and \(x^*-x(t) > 0\), \(H\) is convex in the control variable (\(c\)) and hence, in addition to the state equation (Eq. (2)), the following conditions are necessary and sufficient for minimum lifetime suffering:

\[
\dot{\lambda}(t) = -\frac{\partial H}{\partial x} = g(c(t)) - \lambda(t)r
\]

\[
\frac{\partial H}{\partial c} = g'(c(t))[x^*-x(t)] - \lambda(t)\delta = 0
\]

and the tranversality condition \(\lim_{t \to \infty} \lambda(t)x(t) = 0\).

By differentiating Eq. (5) with respect to time, substituting the right-hand sides of Eq. (4) and Eq. (5) for \(\dot{\lambda}\) and \(\lambda\), collecting terms and multiplying both sides of the

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\(^2\) Concerning suicide, see Hamermesh and Soss (1974).
resultant equation by \(1/g'(c(t))\), the following no-arbitrage rule of lifetime-suffering minimizing drinking is obtained:

\[
\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] + \delta[g(c(t))/g'(c(t))] + [rx(t) - \delta c(t)]}{[g''(c(t))/g'(c(t))][x^* - x(t)]}
\]  

(6)

or equivalently,

\[
\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] + \delta c(t)[1 + 1/\xi(c(t))]}{[g''(c(t))/g'(c(t))][x^* - x(t)]}
\]  

(7)

where

\[
\xi(c(t)) \equiv -g'(c(t)) \cdot \frac{c(t)}{g(c(t))}
\]  

(8)

denotes the elasticity of the instantaneous suffering-moderating drinking.

Recalling assumptions 1, 2 and 5 the denominator is negative and \(A(t)\) and \(B(t)\) are positive. Hence, \(\dot{c}(t) = 0\) as \(A(t) = B(t)\). That is, the likelihood that the consumption of alcohol by a lifetime-suffering minimizing ACL increases from one instance to another:

1. diminishes with the difference between the desired state and current status and proportionally to the difference between the rate of time preference and the rate of return on status,

2. diminishes with the return on the current status,
3. increases with the current consumption of alcohol and its negative adverse effect on status, but in a proportion that is declining with the elasticity of the instantaneous suffering-relief degree of alcohol.

IV. ACL’s Rational Stationary and Non-Stationary Alcohol Consumption and Status in the Absence of Public Reaction

The stationary rational consumption level of alcohol and status are computed for an instantaneous suffering-relieving-drinking function displaying a constant elasticity $\mu$ and satisfying assumption 2: namely,

$$g(t) = e^{-\mu c(t)}.$$  \hspace{1cm}  (9)

By substituting this specification into Eq. (6), the no-arbitrage rule can be rendered as

$$c(t) = \frac{\dot{x(t)}(\rho - r)[x^* - x(t)] + (\delta / \mu) + \left[rx(t) - \delta c(t)\right]}{-\mu[x^* - x(t)]}.$$  \hspace{1cm}  (10)

By substituting the steady-state condition ($x = 0 = c$) into this no-arbitrage rule and the status-motion equation (Eq. (2)), the isocline $c = 0$ is given by

$$(\rho - r)[x^* - x_{ss}] - (\delta / \mu) = 0.$$  \hspace{1cm}  (11)

and the isocline $x = 0$ by

$$c = \left(\frac{r}{\delta}\right)x.$$  \hspace{1cm}  (12)
and displayed by phase-plane diagram in Figure 1. The stationary levels of alcohol consumption and status are
\[ x_{ss} = x^* - \left[ \frac{\delta}{\mu} \right] \]  
(13)
and
\[ c_{ss} = \left( \frac{r}{\delta} \right) x^* - \left[ \frac{r}{\mu} \right]. \]  
(14)

These expressions of the stationary status and alcohol consumption and the phase-plane analysis lead to the following propositions.

**Proposition 1** (Impatience): The ACL’s rate of time preference exceeds the rate of return on his status. That is, \( \rho > r. \) (See Appendix for a proof.)

**Proposition 2:** The gap between the ACL’s desired status and stationary status is narrowed by the difference between his rate of time preference and rate of return on status. This narrowing of the desired status-stationary status gap is strengthened by the ratio of the elasticity of the instantaneous suffering-relief degree (the blessing) to the status-eroding effect of alcohol (the curse). (See Appendix for a proof.)

**Proposition 3:** The ACL’s stationary consumption of alcohol rises with the difference between his rate of time preference and rate of return on status and with the elasticity of the instantaneous suffering-relief degree of alcohol but declines with the status-eroding effect of alcohol. (See Appendix for a proof.)
Proposition 4: The ACL’s stationary consumption of alcohol declines with the rate of return on status if \( \frac{\delta \rho}{\mu (\rho - r)^2} > x^* \). (See Appendix for a proof.)

Proposition 5: If the ACL’s rate of return on his status is larger than half his rate of time preference (i.e., \( r > 0.5 \rho \)), then \((x_{ss}, c_{ss})\) is a saddle point and can (only) be approached along two convergent arms as displayed by Figure 1. Along the lower convergent arm the ACL’s status is improved despite the increase in his alcohol consumption, whereas along the upper convergent arm the ACL’s status is eroded despite the decline in his alcohol consumption. (See Appendix for a proof.)

Insert Figure 1 here

Proposition 6: If the ACL’s rate of return on his status is smaller than half his rate of time preference (i.e., \( r < 0.5 \rho \)), then \((x_{ss}, c_{ss})\) is an asymptotically unstable spiral. (See Appendix for a proof.)

Insert Figure 2 here

V. Society’s Reaction to Alcoholism and its Implication for the ACL’s Rational Alcohol Consumption and Status

So far the analysis has ignored the reaction of society to excessive alcohol consumption. Given that the economic costs and negative externalities of alcoholism are substantial

with alcohol consumption, is \( g(t) = 1/[1 + \gamma c(t)] \), where \( \gamma \) is a positive scalar.
(e.g., Mullahy and Sindelar, 1993 and 1995), it pays for the society to fight chronic alcoholism. The reaction takes several possible forms, including the way families, friends, employers, institutions and the public sector deal with alcoholics. They react to alcoholism, on the one hand, by supporting the alcoholic psychologically, financially, medically and socially, and, on the other hand, by punishing heavy drinking through the creation of restrictions on alcohol consumption. In terms of the model, the reaction and intervention of society improves the alcoholic individual status:

\[ x(t) = r x(t) - \delta c(t) + \beta E(t). \]  

(15)

where \( \beta \) is a positive scalar and \( E(t) \) represents the various ways society helps ACLs.

Taking the ACL as a representative alcoholic, the society’s level of reaction to and intervention in the ACL’s behavior increases with his level of alcohol consumption:

\[ E(t) = \alpha c(t) - \Omega \]  

(16)

where \( \alpha \) and \( \Omega \) are positive scalars. The parameter \( \Omega \) stands for the maximal level of alcohol consumption tolerated by society. Of course abusive alcohol consumption is culturally variable. According to Vaillant (1983) the consumption of alcohol that would be acceptable in one culture may be considered alcoholism in another. The effect of society’s reaction changes substantially the ACL’s model presented in the previous sections.

The modified ACL model assumes that the rational ACL anticipates the society’s reaction to his alcohol consumption and incorporate this reaction into his selection of the suffering-relieving alcohol consumption trajectory. In formal terms, he chooses \( c \) so as to

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4 Hamilton and Hamilton (1997) for Canada and Barrett (2002) for Australia found that moderate drinkers received a wage premium relative to non-drinkers while heavy drinkers received a substantial wage penalty.
minimize his lifetime suffering, $\int_{0}^{\infty} e^{-\gamma t} g(c(t))[x^*-x(t)]dt$, subject to his actual status’
evolution and the society’s reaction and intervention as described by Eq. (15) and Eq. (16).

The Hamiltonian corresponding to this problem is:

$$H(t) = g(c(t))[x^*-x(t)] + \lambda(t)[rx(t) - \delta c(t) + \beta E(t)] + \theta(t)[\alpha c(t) - \Omega]$$  \hspace{1cm} (17)

where the costate variable $\theta(t)$ indicates the shadow price of the society’s reaction at $t$.

The first-order conditions are:

$$\frac{\partial H}{\partial c} = g'(c(t))[x^*-x(t)] - \lambda(t)\delta + \theta(t)\alpha = 0$$  \hspace{1cm} (18)

$$\dot{\lambda}(t) - \rho \dot{\lambda}(t) = - \frac{\partial H}{\partial x} = g(c(t)) - \lambda(t)r$$  \hspace{1cm} (19)

$$\dot{\theta}(t) - \rho \dot{\theta}(t) = - \frac{\partial H}{\partial E} = -\lambda(t)\beta.$$  \hspace{1cm} (20)

and the tranversality conditions $\lim_{t \to \infty} \lambda(t)x(t) = 0 = \lim_{t \to \infty} \theta(t)E(t)$

The interesting result of this model is that it can generate complex dynamics to alcohol consumption, such as persistent cyclical paths. This result contrasts with the explosive oscillations found in Proposition 6, because the long-run equilibrium is not a single point, but rather an invariant manifold [i.e. the limit cycle]. Cyclical paths must not come as a surprise since the literature has examined cycles in similar contexts. The theory of rational addiction, for instance, is capable of explaining cyclical consumption paths expressed as damped or explosives waves or limit cycles (e.g., Dockner and Feichtinger, 1993). In the same vein, medical treatment of chronic diseases, like diabetes, creates incentives to make food consumption and labor supply display permanent cyclical patterns.
as well (e.g., Faria, 2003). Permanent cycles in alcohol consumption seem to be empirically significant as well. Empirical studies (e.g., Kerr et al., 2002) suggest that there appear to be important subgroups of alcohol drinkers that move between abstention and light drinking and moderate and heavy drinking.

In order to examine the possibility of such complex behavior let us use Eq. (18) to express alcohol consumption as a function of \( \lambda, \theta \), and \( x \):

\[
g'(c(t))[x^*-x(t)] = \lambda(t)\delta - \theta(t)\alpha \Rightarrow c(t) = c(x(t), \lambda(t), \theta(t)) .
\]  

(18’)

By differentiation, the following properties are obtained.

\[
c_x = \frac{g'(c(t))}{g''(c(t))[x^*-x(t)]} < 0; \quad c_\lambda = \frac{\delta}{g''(c(t))[x^*-x(t)]} > 0; \quad c_\theta = \frac{-\alpha}{g''(c(t))[x^*-x(t)]} < 0 .
\]

The substitution of equation (18’) into equations (15), (16), (19), (20) yields:

\[
x(t) = r x(t) - \delta c(x(t), \lambda(t), \theta(t)) + \beta E(t)
\]  

(15’)

\[
E(t) = \alpha c(x(t), \lambda(t), \theta(t)) - \Omega
\]  

(16’)

\[
\lambda(t) = [\rho - r]\lambda(t) + g(c(x(t), \lambda(t), \theta(t)))]
\]  

(19’)

\[
\theta(t) = \rho \theta(t) - \lambda(t) \beta
\]  

(20’)

With this presentation of the first-order conditions, the condition for a limit cycle between \( x \) and \( E \), and, consequently (through Eq. (18’)) for cyclical alcohol consumption can be obtained.

**Proposition 7**: If \( r > \rho > 0 \), there is a limit cycle between \( x \) and \( E \). (See Appendix for a proof.)
Corollary: If \( x \) displays cyclical behavior, then alcohol consumption is cyclical. (See Appendix for a proof.)

The permanent cyclical path of alcohol consumption derived above relies on the case that the rate of return of status, \( r \), exceeds the rate of time preference, \( \rho \). It has been assumed that both terms are positive parameters. In contrast to Proposition 7, Proposition 1, derived from the basic ACL model, precludes the case of \( r > \rho \) because in the context of the basic ACL model it violates Assumption 1 (namely, imminent failure). As stressed in the beginning of this section, the inclusion of society’s reaction to alcoholism changes the basic ACL model substantially. In the modified ACL model Proposition 1 no longer holds, and the condition for cyclical consumption (i.e., \( r > \rho > 0 \)) does not violate assumption 1. This is because the steady-state value of \( x \), denoted as \( \bar{x} \), in the modified ACL model is given by:

\[
\bar{x} = x^* - \frac{\alpha \beta - \delta \rho}{\rho (\rho - r)}
\]

which is different from the steady-state value displayed by Eq. (13). In addition, note that there is no overshooting when \( \alpha \beta < \delta \rho \); or, equivalently, as long as the individual’s rate of time preference exceeds the ratio of the effectiveness of the society’s remedial measures to the status-eroding effect of alcohol (i.e., \( \rho > \alpha \beta / \delta \)).

The importance of the cyclical pattern for alcohol consumption is clear. When society reacts by trying to improve the life of an ACL, by providing him help and support and punishing excessive drinking, the ACL can behave in a cyclical manner. That is, he can show signs of improvement by reducing his alcohol consumption for a while, which makes
social pressure decrease. As a consequence, when social pressure decreases the ACL increases its alcohol intake up to the point in which his status declines and forces the society to intervene making him reduce his consumption once again. Anecdotal evidence from clinics specializing in the treatment of alcoholics and other drug addicts shows that this pattern is common for patients with recurrent interventions.

VI. Concluding Remarks

This paper diverges from the utility-maximizing paradigm, which has been used to study alcohol and other types of addiction, to provide a conceptual framework for analyzing the phenomenon of suffering minimization and the persistent consumption of alcohol. It assumes that present alcohol consumption alleviates the individual’s current suffering, but leads to future failures and greater potential suffering. The paper employs a dynamic framework that incorporates this inter-temporal trade off, considers the chronic loser’s suffering and alcohol consumption over his lifetime, and takes the alcohol consumption path which minimizes the chronic loser’s lifetime suffering stemming from the distance between his desired status and actual status to be rational.

The basic model shows that the stationary status of the alcoholic chronic loser is improved by the difference between his rate of time preference and the rate of return on his status and that this improvement is amplified by the ratio of the effectiveness of alcohol in reducing instantaneous suffering (the blessing) to the status-eroding effect of alcohol (the curse). However, when society’s reaction to alcoholism is taken into account, the model is substantially altered. Society affects alcohol consumption by providing, on the one hand, help and support to heavy drinkers and, on the other hand, creating mechanisms
to refrain from alcohol consumption. It is shown that the rate of return on alcoholic’s status can be greater than his time preference. This situation can make alcohol consumption display complex dynamics, such as permanent cyclical paths.
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APPENDIX: Proofs of Propositions and Corollary

Proof of Proposition 1: By virtue of Eq. (13), \( x_{ss} \geq x^* \) when \( \rho \leq r \). However, accurately and overshooting are not compatible with assumption 1 and with minimizing lifetime suffering. \textit{QED}

Proof of Proposition 2: Straightforward from Eq. (13). \textit{QED}

Proof of Proposition 3: Straightforward from Eq. (14). \textit{QED}

Proof of Proposition 4: By differentiating Eq. (14) with respect to \( r \). \textit{QED}

Proof of Proposition 5: By differentiating Eq. (2), \( \frac{dx}{dc} = -\delta < 0 \), and therefore the horizontal arrows above (below) the isocline \( \dot{x} = 0 \) are leftward (rightward) directed. By differentiating the Eq. (10) with respect to \( x \)

\[
\frac{dc}{dx} = \frac{\delta \mu + \delta - r \mu x}{\mu^2 (x^* - x)^2} = \frac{\delta \mu[c + (1/\mu) - (r/\delta)x^*]}{\mu^2 (x^* - x)^2}.
\]

Hence, \( \frac{dc}{dx} < 0 \) at the vicinity of the stationary point if \( c_{ss} < (r/\delta)x^* - (1/\mu) \). Recalling Eq. (14),

\[
c_{ss} = \left( \frac{r}{\delta} \right)x^* - \left( r/\mu(\rho - r) \right) < \left( \frac{r}{\delta} \right)x^* - \frac{1}{\mu} \quad \text{if} \quad [r/\mu(\rho - r)] > 1/\mu \quad \text{or, equivalently, if}
\]

\[ [r/(\rho - r)] > 1. \] This in turn implies that \( \frac{dc}{dx} < 0 \) at the vicinity of the stationary point if \( r > 0.5 \rho \). In this case, the vertical arrows are downward (upward) directed in the region on the right (left) hand side of the isocline \( \dot{c} = 0 \). These directions of the horizontal and vertical arrows at the vicinity of the steady state implies that \( (x_{ss}, c_{ss}) \) is a saddle point and can be approached along the two convergence arms as displayed by Figure 1. \textit{QED}
Proof of Proposition 6: As in the proof of proposition 5, but given that \( r < 0.5 \alpha \) then

\[
\frac{dc}{dx} < 0.
\]

In this case, the vertical arrows are upward (downward) directed in the region on the right (left) hand side of the isocline \( \dot{c} = 0 \). These directions of the horizontal and vertical arrows at the vicinity of the steady state implies that \((x_{ss}, c_{ss})\) is either a spiral or a center. The linearization of the differential equation system consisting of Eq. (2) and Eq. (10) reveals that the trace of the state-transition matrix is \( r + \delta / (x^* - x) \). Recalling assumptions 1, 2 and 5, \( r + \delta / (x^* - x) > 0 \). That is, the real part of the conjugate-complex characteristic roots of the state-transition matrix is positive and hence \((x_{ss}, c_{ss})\) is an asymptotically unstable spiral. \( QED \)

Proof of Proposition 7: Following Feichtinger et al. (1994), in order to prove the existence of a limit cycle it is necessary to show that the signs of the determinant of the Jacobian, \( |J| \), of the system (15’), (16’), (19’) and (20’) given by:

\[
|J| = \begin{vmatrix}
\frac{\partial x}{\partial x} & \frac{\partial x}{\partial E} & \frac{\partial x}{\partial \lambda} & \frac{\partial x}{\partial \theta} \\
\frac{\partial x}{\partial x} & \frac{\partial E}{\partial E} & \frac{\partial E}{\partial \lambda} & \frac{\partial E}{\partial \theta} \\
\frac{\partial \lambda}{\partial x} & \frac{\partial \lambda}{\partial E} & \frac{\partial \lambda}{\partial \lambda} & \frac{\partial \lambda}{\partial \theta} \\
\frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial E} & \frac{\partial \theta}{\partial \lambda} & \frac{\partial \theta}{\partial \theta}
\end{vmatrix} \quad (A7.1)
\]

and the term \( M \), defined below:
\[ M = \begin{bmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial \lambda} \\ \frac{\partial E}{\partial x} & \frac{\partial E}{\partial \lambda} \end{bmatrix} + \begin{bmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial \theta} \\ \frac{\partial E}{\partial x} & \frac{\partial E}{\partial \theta} \end{bmatrix} + 2 \begin{bmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial \lambda} \\ \frac{\partial E}{\partial x} & \frac{\partial E}{\partial \lambda} \end{bmatrix} \]  

(A7.2)

are positive when calculated with the steady-state levels \((\bar{x}, \bar{E}, \bar{\lambda}, \bar{\theta})\). Furthermore, the value of the bifurcation parameter \((\rho)\) given by the condition below:

\[ |J| = \left( \frac{M}{2} \right)^2 + \rho^2 \left( \frac{M}{2} \right) \]

(A7.3)

must be positive as well. These are the conditions for the existence of a limit cycle according to the Hopf bifurcation theorem [e.g., Guckenheimer and Holmes, 1990]. One can verify that

\[ |J| = -\beta [\alpha \gamma, \rho (p - r) g' c_k - \rho \alpha g' c_k c_j] > 0 \iff r > \rho \]

and

\[ M = (r - \delta c_k) (p - r) g' c_k + g' c_k c_k \delta + 2 \beta g' c_0 > 0 \iff r > \rho . \]

Consequently, it follows by equations (A7.1), (A7.2) and (A7.3) that the necessary and sufficient condition for the existence of a limit cycle between \(x\) and \(E\) is \(r > \rho > 0\). \textit{QED}

Proof of Corollary: The cyclical behavior of \(c\) follows from Eq. (18').

\[ ^5 \text{The steady state equilibrium is found when } \dot{\lambda} = \dot{\theta} = \dot{x} = \dot{E} = 0. \]
Figure 1. Rational status and alcohol consumption when $r > 0.5p$

Figure 2. Rational status and alcohol consumption when $r < 0.5p$