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GREENING THE NEOCLASSICAL OPTIMAL ECONOMIC GROWTH MODEL

by

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Abstract

A no-arbitrage rule of consumption and a golden rule of capital accumulation are derived under the assumptions that the satisfaction from consumption is spoiled by environmental degradation caused by industrialisation but moderated by cleaning up and greening operations.

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1. Introduction

The neoclassical optimal growth model constructed by Ramsey (1928) and refined by Cass (1965), Koopman (1965) and Phelps (1966) is extended to include environmental aspects. The extension is based on two premises. The first is that, unable to migrate, the instantaneous satisfaction of the representative agent from consumption is spoiled by the rate of degradation of her home physical environment. The second is that the rate of environmental degradation rises with the level of industrialisation of the economy, but in a degree moderated by cleaning up and greening operations. These premises are formalised and integrated into the neoclassical optimal growth model in section 2. The properties of the resultant environment-Augmented no-arbitrage rule of consumption and golden rule of capital accumulation are discussed in section 3. The model is further extended in section 4 to include a natural rehabilitation process.

2. The environment-augmented growth model

It is assumed that the instantaneous satisfaction (u) of the representative agent from consumption (c) is depreciated by the rate of degradation ($0 \le x \le 1$) of her physical environment. That is,

$$\tilde{u}(t) = [1 - x(t)]u(c(t)) \quad (1)$$

where, u is increasing but concave in c, $\tilde{u} = u$ when $x = 0$ (i.e., pristine environment) and $\tilde{u} = 0$ when $x = 1$ (i.e., completely degraded environment that cannot sustain human life). It is assumed that, although some of her members of society can migrate (e.g., the more daring, the more talented and the younger), the representative agent is confined to her initial location.

It is also assumed that the rate of environmental degradation rises with the level of industrialisation of the economy, represented by the capital-labor ratio (k), but in a degree moderated by the per capita (i.e., the representative agent's) spending on cleaning up and greening operations (g):

$$\dot{x}(t) = [g_0 - g_1 g(t)]k(t) \quad (2)$$

where g_0 is a positive scalar denoting the adverse marginal effect of industrialization on the environment when cleaning up and greening operations are not implemented, and g_1 is a positive scalar representing the marginal moderating effect of cleaning up and greening operations on the rate of degradation of the environment. In this context, $[g_0 - g_1 g(t)]$ represents the environment degradation factor. When the representative agent's spending on cleaning up and greening operations rises with her income and when the cleaning up and greening operations have a considerable moderating marginal effect on the environment degradation factor, so that for a sufficiently large clean-up spending the environment degradation factor is negative, this motion equation is consistent with the environmental Kuznets curve hypothesis of an inverted U-shaped relationship between the quality of the environment and per capita income (Shafik and Bandyopadhyay, 1992; Seldon and Song, 1994; Grossman and Krueger, 1995).

The representative agent's spending on cleaning up and greening operations modifies the conventional motion equation of her capital stock as follows:

$$\dot{k}(t) = f(k(t)) - c(t) - g(t) - (d + n)k(t)\,. \quad (3)$$

where f is a concave function yielding the per capita income, d is the capital depreciation rate, and n is the population growth rate.

By incorporating these premises into the standard neoclassical optimal growth model, the problem of the representative agent having a fixed time-preference rate r can be presented as choosing the joint trajectories of consumption and cleaning-up and greening spending so as to maximize her sum of discounted instantaneous utilities

$$V = \int_0^\infty e^{-rt}[1 - x(t)]u(c(t))dt \quad (4)$$

subject to the change in the quality of her environment (equations (2)) and the change in her capital stock (equation (3)). Following Kamien and Schwartz (1991, Section 9), if

$[1-x(t)]$ is interpreted as the probability of living beyond t, V is the expected lifetime utility of the representative agent whose life expectancy is uncertain and endangered by environmental degradation.

3. The environment-augmented no-arbitrage and golden rules

The solution to this optimal control problem leads to the following no-arbitrage rule:

$$\dot{c}(t)=\frac{f'(k)-(r+d+n)}{-u''(c(t))/u'(c(t))}-\frac{\left[\frac{1}{g_1 k(t)}+\frac{k(t)}{1-x(t)}\right][g_0-g_1 g(t)]}{-u''(c(t))/u'(c(t))}. \qquad (5)$$

(See the Appendix for a detailed derivation.)

As it is appealing to analyze the rate of change of consumption, the following convenient specification of the instantaneous utility function is considered

$$u=c^b \qquad (6)$$

where $0<b<1$. In this case, the environment-augmented no-arbitrage rule (5) of consumption, can be rendered as

$$\frac{\dot{c}(t)}{c(t)}=\underbrace{\left(\frac{1}{1-b}\right)[f'(k)-(r+d+n)]}_{A}-\underbrace{\left(\frac{g_0-g_1 g(t)}{1-b}\right)\left[\frac{1}{g_1 k(t)}+\frac{k(t)}{1-x(t)}\right]}_{B} \qquad (7)$$

The first term (A) on the right-hand side of equation (7) is the conventional no-arbitrage rule of consumption obtained by the aforementioned optimal growth literature. It states that the instantaneous change in consumption corresponds to the difference between the marginal product and the user cost of capital, discounted by the degree of concavity of the instantaneous utility function. The second term (B) modifies this conventional no-arbitrage rule for the case where environmental aspects are taken into account.

Recalling that $Sign(B) = Sign[g_0 - g_1 g(t)]$, the environment-augmented no-arbitrage rule (7) recommends a smaller, the same, or a larger rate of consumption change than that advocated by the conventional no-arbitrage rule as the optimal spending on cleaning up and greening operations is smaller than, equal to or greater than the stationary level g_0 / g_1, respectively. However, this environment-augmented no-arbitrage rule does not enable an analytical derivation of the optimal relationship between the quality of the representative agent's environment ($1-x$) and her income ($f(k)$) which might, or might not coincide, with the ad hoc environmental Kuznets curve hypothesis.

The no-arbitrage rule (7) indicates further that the change in the quality of the environment induced by the accumulation of capital makes the difference between the capital-labour ratios in the environment-augmented optimal growth model and the conventional optimal growth model, except in steady state. By setting $\dot{c}$ to be equal to zero, the stationary capital-labor ratio satisfies

$$f'(k_{ss}) = (r + d + n) + [g_0 - g_1 g_{ss}]\left[\frac{1}{g_1 k_{ss}} + \frac{k_{ss}}{1 - x_{ss}}\right] \quad (8)$$

where the subscript ss denotes steady state levels. Recall that $g_{ss} = g_0/g_1$, the ratio of the marginal effect of industrialization on the environment when cleaning up and greening operations are not implemented and the marginal moderating effect of cleaning up and greening operations on the rate of degradation of the environment. Consequently, the second term on the right-hand-side is equal to zero and the stationary capital-labor in the proposed environment-augmented growth model satisfies the familiar golden rule of equality between the marginal product of per capita capital and its user cost – the sum of the rates of time preference, capital depreciation and population growth. That is, as in the standard neoclassical optimal growth model

$$k_{ss} = f'^{-1}(r + d + n)\,. \tag{9}$$

4. Natural rehabilitation

If there exists a natural environmental rehabilitation process the environment-augmented stationary capital-labour ratio may differ from the stationary capital-labour ratio advocated by the conventional golden rule. For simplicity, let the natural rehabilitation rate be constant: namely, a positive scalar m. By subtracting $mx(t)$ from the right-hand-side of the environment degradation motion equation (2), the no-arbitrage rule is now rendered as

$$\frac{\dot{c}(t)}{c(t)} = \underbrace{\left(\frac{1}{1-b}\right)[f'(k) - (r + d + n)]}_{A} - \underbrace{\left(\frac{g_0 - g_1 g(t)}{1-b}\right)\left[\frac{1}{g_1 k(t)} + \frac{k(t) - mx(t)}{1 - x(t)}\right]}_{B'} \tag{10}$$

and the spending on cleaning up and greening operation in steady state declines with the ratio of the stationary degradation rate to the stationary per capita capital:

$$g_{ss} = \frac{g_0}{g_1} - \left(\frac{m}{g_1}\right)\frac{x_{ss}}{k_{ss}}\,. \tag{11}$$

Consequently, the stationary capital-labor ratio in this environment-augmented optimal growth model satisfies

$$f'(k_{ss}) = (r + d + n) + m\frac{x_{ss}}{k_{ss}}\left[\frac{1}{g_1 k_{ss}} + \frac{k_{ss} - mx_{ss}}{1 - x_{ss}}\right]. \tag{12}$$

The golden (i.e., stationary) capital-labor ratio in this version of the environment-augmented optimal growth model is equal to the golden capital-labor ratio in the standard neoclassical optimal growth model if and only if

$$\frac{k_{ss}}{x_{ss}} = m - \frac{1 - x_{ss}}{g_{1k_{ss}} x_{ss}}. \tag{13}$$

Recalling that f is concave, the golden, environment-augmented, capital-labor ratio is smaller (larger) than the golden capital-labor ratio in the standard neoclassical optimal growth model if $\frac{k_{ss}}{x_{ss}}$ is larger (smaller) than $m - \frac{1 - x_{ss}}{g_{1k_{ss}} x_{ss}}$.

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Appendix

The Hamiltonian associated with the optimal control problem is

$$H = e^{-rt}(1-x)u(c) + l_1[f(k) - c - g - (d+n)k] + l_2(g_0 - g_1 g)k \qquad \text{(A1)}$$

where l_1 and l_2 are the shadow values of the representative agent's capital and rate of environmental degradation, respectively, and where the time index t is omitted for convenience.

The adjoint equations are:

$$\dot{l}_1 = -\frac{\P H}{\P k} = -l_1[f'(k) - (d+n)] - l_2(g_0 - g_1 g) \qquad \text{(A2)}$$

and

$$\dot{l}_2 = -\frac{\P H}{\P x} = e^{-rt}u(c)\,. \qquad \text{(A3)}$$

The optimality conditions are:

$$\frac{\P H}{\P c} = e^{-rt}(1-x)u'(c) - l_1 = 0 \qquad \text{(A4)}$$

and

$$\frac{\P H}{\P g} = -l_1 - l_2 g_1 k = 0\,. \qquad \text{(A5)}$$

By differentiating the optimality condition (A4) with respect to t the following singular control equation is obtained:

$$-e^{-rt}u'(c)\dot{x} - re^{-rt}(1-x)u'(c) + e^{-rt}(1-x)u''(c)\dot{c} - \dot{l}_1 = 0 \,. \qquad \text{(A6)}$$

By substituting equations (A2), (A4) and (A5) into equation (A6) for $\dot{l}_1$, l_1 and for l_2, respectively, and rearranging terms, the no-arbitrage rule (5) is obtained.