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Behaviour and the Prevalence of AIDS**

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**Abstract**

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## A Lifetime Portfolio of Risky and Risk-Free Sexual Behavior and the Prevalence of AIDS

*A lifetime portfolio of risky and risk-free sexual activities is conceptually constructed in this paper. People's time allocation between risky and risk-free sexual activities affects, and is affected by, the prevalence of AIDS. A small satisfaction differential between risky sex and risk-free sex can lead to a significant prevalence of AIDS. Numerical simulations suggest that the reduction in the prevalence of AIDS generated by a one percent improvement in the sensual quality of freely distributed condoms can be 0.855 percent when the initial satisfaction differential between risky sex and risk-free sex is 50 percent, or 0.464 percent when the initial satisfaction differential is 100 percent. (JEL I19, J17, D91)*

Since the eruption of the Acquired Immune Deficiency Syndrome (AIDS) in June 1981 seventeen million sub-Saharan Africans succumbed to the disease. Twenty-five million sub-Saharan Africans are currently infected with AIDS. For a population of half a billion people these figures are staggering. The epidemic has not been confined to sub-Saharan Africa. Eighteen million people have been infected with AIDS in the rest of the world, of whom five millions have already died. In total, during the first twenty years of the pandemic one percent of the world population (about the population of Great Britain) has been infected with the AIDS virus and 0.37 percent (nearly the population of Scandinavia) has succumbed to it. The level of devastation will be immensely increased when the epidemic reaches a take-off stage in the heavily populated, poor and socially and politically unstable central, southern and eastern regions of Asia.<sup>1</sup>

Only five percent of the people currently infected with AIDS in sub-Saharan Africa are identified. The rest, about twenty-four million, continue fuelling the epidemic in one of the world's poorest and socially and politically unstable regions. A severe identification problem also exists in all the other parts of the world. While tests for AIDS can be inexpensively provided, medical care is expensive. Twenty billion dollars are needed every year for granting a proper medical care for the AIDS sufferers of sub-Saharan Africa alone—about three times the nominal per capita income in that part of the world for any infected person. In the absence of adequate private resources, strong governmental

commitment, coordination and funding, and massive international assistance, there is a low incentive for the people of sub-Saharan Africa to participate in tests for AIDS. Fearful of stigmatism and ostracism if tested positive, they avoid the tests. Even if the test results were confidential, in the absence of significant help and medical care many may prefer illusion to realisation.<sup>2</sup>

Although the AIDS virus can also be transmitted from mother to fetus and by contaminated blood transfusions and sharing of hypodermic needles, the present paper focuses on the transmission of AIDS through sexual activity. People's interest and participation in sex exceed the level needed for reproduction and, despite the risk of contracting AIDS, the marriage's confines. The probability of contracting AIDS and the prevalence of the disease can be limited by using a simple device—a condom. In developed countries condoms can be bought in supermarkets for a few cents per unit, and in many developing areas the distribution of condoms is highly subsidised. Yet condoms are not always used in non-reproductive sexual activities and not necessarily due to a high price (relative to income) and uncertain supply (Philipson and Posner, 1995). In view of the costs of bearing the risk of contracting AIDS and that in many places condoms are cheaply and steadily supplied, it must be the case that people derive considerably higher level of satisfaction from sexual intercourse unrestrained by condoms.

Sexual intercourse restrained by condoms are considered in this paper to be risk-free. In the absence of perfect information about the identity of those infected with AIDS and self HIV-AIDS status, which is the case for many people, sub-Saharan Africans in particular, unrestrained sex is considered to be risky. The transmission of AIDS through risky sexual intercourse cannot be solely attributed to erratic and uncalculated behavior. Analogically to the construction of efficient portfolios and pricing of risky assets in the presence of a risk-free asset,<sup>3</sup> rational people incorporate the trade off between the extra satisfaction from having unrestrained sex and the associated costs of risk bearing into their decision-making process and may prefer a portfolio of risky sex and risk-free sex to a corner solution of exclusive engagement in risk-free sex.<sup>4</sup>

An alternative, no less appealing, interpretation of the portfolio framework adopted by this paper is to consider the population as a whole and divide it, similarly to Kremer

(1996), into two groups: highly sexually active people and, slightly different from Kremer (*ibid*), sexually conservative people. From the point of view of the society as a whole, the first group is a risky human asset and the second is a risk-free human asset. Both assets are engaged in the aggregate production of sexual satisfaction. At any point in time, the expected contribution of a unit of the risky group to the instantaneous aggregate level of sexual satisfaction exceeds the expected contribution of a unit of the risk-free group, but at a cost of increased uncertainty about the society's quality and expectancy of life. The relative size of the groups evolves over time in accordance with the prevalence of AIDS and other utilitarian, demographic and epidemiological factors. In turn, the prevalence of the disease is affected by the relative size of these groups. In view of this interpretation, the portfolio model developed in this paper not only incorporates, but also endogenizes, Kremer's (*ibid*) population decomposition and its cause and effect relationship with the prevalence of AIDS.

The paper analyses the prevalence of AIDS in an uncoordinated group of people who maximize their expected lifetime-utility from sex. In view of the adverse implications of AIDS on life quality and duration, we refer to the members of this group as rational.<sup>5</sup> The analysis focuses on a representative member of this group who, like in the case of most sub-Saharan Africans, does not know the identity of the group members infected with AIDS as well as his, or her, own condition. The analysis of the representative member's sexual behavior incorporates three factors: inducement, risk and transmission. It shows that, in addition to having risk-free sex, he, or she, may choose to be engaged in the risky, but more satisfying, sexual activity. He, or she, might be added to the list of the epidemic's casualties and might also contribute to its transmission. It is shown that, under sensible choice of the model parameter values, not much inducement is needed for a sexually transmitted disease to reach a level of epidemic in an uncoordinated group of rational people.

The paper is structured as follows. Section I presents the building blocks of the rational representative member's construction of dynamic portfolio of risk-free and risky sex. The characteristics of his, or her, efficient-portfolio construction are summarised in section II by a set of necessary conditions on the privately optimal allocation of time between risk-

free and risky sexual activities over the lifespan. The implications of the representative member's privately optimal time allocation between risk-free and risky sexual activities for the stationary level of the prevalence of AIDS in his, or her, group are analysed in section III. Numerical simulations reveal the possible effects of inducement, risk, transmission, time preference, and life-expectancy aspects on the stationary prevalence of the disease. Trajectories of the representative member's intensity of risky sexual activity and the associated prevalence of the disease in the group are displayed in a phase-plane diagram in section IV for alternative initial conditions. While the previous sections analyze the intensity of risky sex and the prevalence of AIDS under time-separable preferences and fixed inducement level, section V considers the representative member's preference to be time-non-separable and allows his, or her, inducement level to evolve in accordance with past intensities of risky and risk-free sexual behavior. Section VI concludes.

### I. Building blocks of the rational representative member's sexual portfolio<sup>6</sup>

Suppose that the representative member's lifetime-utility function is additively separable and that his, or her, instantaneous utility,  $u$ , increases with both risk-free and risky sexual activities. For any given intensity, the marginal instantaneous utility from risky sex exceeds the marginal instantaneous utility from risk-free sex. Assuming that condoms are readily and freely available, there is no other justification for bearing the risk associated with unrestrained sex.<sup>7</sup> We refer to this positive marginal instantaneous-utility differential between risky sex and risk-free sex as *the inducement factor*.

Denoting the portion of the time allocated to risky sex by  $x$  ( $0 \leq x \leq 1$ ), the portion of the time allocated to risk-free sex by  $1 - x$ , and the relative effectiveness of risky sex by a scalar  $\alpha > 1$ , the utility from sex at time  $t$  is assumed to be given by

$$u(t) = u(\alpha x(t) + (1 - x(t))) = u(1 + (\alpha - 1)x(t)). \quad (1)$$

As suggested in the introduction,  $x$  can be alternatively interpreted as the population share of the highly sexually active people,  $1-x$  as the population share of the sexually conservative people and  $u$  the society's instantaneous satisfaction level from sex.

Suppose further that the representative member's rate of time preference is a non-negative scalar  $\rho$ , that there is an upper-bound,  $T$ , on his, or her, life expectancy, and that his, or her, probability of dying at any instant  $t$  is  $0 < p(t) < 1$  for  $t < T$ . Being aware of the uncertainty about his, or her, life expectancy that can be induced by engagement in unrestrained sex and the prevalence of AIDS in his, or her, reference group and aging, our expected-lifetime-utility-maximizing representative member multiplies his, or her, accumulated utility from sex between the starting point of the

planning horizon, 0, to a possible time of death  $t$ ,  $\int_0^t e^{-\rho\tau} u(\alpha x(\tau) + (1-x(\tau))) d\tau$ , by

the probability of dying at that instant  $t$ ,  $p(t)$ . The products of  $p(t)$  and

$\int_0^t e^{-\rho\tau} u(\alpha x(\tau) + (1-x(\tau))) d\tau$  associated with any possible life expectancy  $0 \leq t \leq T$

are considered by our rational representative member. The sum of all these products is his, or her, expected lifetime-utility from sex. It is given by the following double-integral expression

$$J = \int_0^T p(t) \left\{ \int_0^t e^{-\rho\tau} u(\alpha x(\tau) + (1-x(\tau))) d\tau \right\} dt. \quad (2)$$

Integrating by parts,<sup>8</sup> our representative member's expected lifetime-utility can be equivalently rendered by a more manageable function

$$J = \int_0^T e^{-\rho t} u(\alpha x(t) + (1-x(t))) \Phi(t) dt \quad (3)$$

where  $\Phi$  is equal to 1 minus the cumulative density function associated with  $p$  and thus indicates the probability of living beyond  $t$ .

In addition to the adverse effect of aging, our representative member's prospects (or the society's prospects if  $x$  is interpreted as the population share of highly sexually active people) of living beyond  $t$  is reduced by the interaction between his, or her, intensity of engagement in risky sexual activity and the prevalence of AIDS in his, or her, group. We denote the proportion of the group infected with AIDS by  $s$  ( $0 \leq s \leq 1$ ) and refer to the adverse effect of this interaction,  $xs$ , on our representative member's prospects of survival as *the risk factor*. Correspondingly, we specify our representative member's probability of living beyond  $t$  as

$$\Phi(t) = [1 - \beta x(t)s(t)]e^{-\mu t} \quad (4)$$

where  $\mu > 0$  reflects the age-effect coefficient,<sup>9</sup> and  $0 \leq \beta < 1$  is the risk-factor coefficient, which is moderated by medical care and the availability of vaccines and is equal to zero, or almost zero, when a perfect vaccine is available.

Substituting equation (4) into equation (5), our representative member's expected lifetime-utility can be now expressed as

$$J = \int_0^T e^{-(\rho + \mu)t} [1 - \beta x(t)s(t)] u(\alpha x(t) + (1 - x(t))) dt. \quad (5)$$

In this framework, the instantaneous utility from consumption is not only discounted by the representative member's degree of impatience ( $\rho$ ), but also by the risk to his, or her, life stemming from aging and engagement in risky sex. Furthermore, by interpreting  $x(t)s(t)$  as the probability of contracting AIDS at  $t$ ,  $\beta x(t)s(t)$  can be alternatively interpreted as the potential deterioration in the quality of the representative member's life

stemming from AIDS and  $\beta$  as the expected rate of deterioration in his, or her, quality of life.

While the proportion of the group infected with AIDS is reduced by attrition, it is increased by the current transmission of AIDS to formerly uninfected members who are presently engaged in risky sexual activity. Similarly to Anderson and May (1991), the net change in the prevalence of the disease within the group is presented for tractability as:

$$\dot{s}(t) = \gamma x(t) - \delta s(t) \quad (6)$$

where  $0 < \gamma < 1$  is the AIDS-transmission coefficient and  $0 < \delta < 1$  is the AIDS-attrition coefficient.<sup>10,11</sup>

The joint operation of the model's components can be illustrated with the complacency assertion: namely, the claim that the recently introduced drug cocktails have an adverse effect on containing the AIDS epidemic in the technologically advanced countries where these cocktails are available and affordable. In terms of our model, the introduction of the drug cocktails reduces the risk-factor coefficient ( $\beta$ ). This leads to a hike in the intensity of the more satisfying, but risky, sexual activity (or to a hike in the population share of the highly sexually active people, under the alternative interpretation of  $x$ ) and, subsequently, to an increase in the prevalence of AIDS ( $s$ ) above the level expected, *ceteris paribus*, from the decline in the AIDS-attrition coefficient ( $\delta$ ).

## II. Conditions for individually optimal portfolio of risk-free and risky sex

Our rational representative member chooses the trajectory of  $x$  that maximizes  $J$  subject to the aforementioned motion equation of  $s$ . To simplify the mathematical analysis we assume that his, or her, instantaneous utility function is linear in the intensities of risky and risk-free sex:

$$u(t) = \alpha x(t) + (1 - x(t)) = 1 + (\alpha - 1)x(t) \quad (7)$$

where  $\alpha - 1$  is the inducement factor.

Subsequently,

$$J = \int_0^T e^{-(\rho+\mu)t} [1 - \beta x(t)s(t)][1 + (\alpha - 1)x(t)] dt. \quad (8)$$

As  $J$  is a concave function of  $x$ , there exists an interior solution to the representative member's optimal-control problem. The Hamiltonian corresponding to his, or her, problem is

$$H(t) = e^{-(\rho+\mu)t} [1 - \beta x(t)s(t)][1 + (\alpha - 1)x(t)] + \lambda(t)[\gamma x(t) - \delta s(t)] \quad (9)$$

where  $\lambda$  is a co-state variable indicating his, or her, (shadow) discontent with the prevalence of AIDS in the group.

In addition to the motion equation (6), the solution trajectories of the representative member's intensity of risky sex and the prevalence of AIDS in the group should satisfy the adjoint equation, which describes the evolution of the representative member's discontent with the prevalence of AIDS,<sup>12</sup>

$$\dot{\lambda} = -\frac{\partial H}{\partial s} = e^{-(\rho+\mu)t} [\beta x + (\alpha - 1)\beta x^2] + \lambda \delta \quad (10)$$

and the optimality condition, which requires that the representative member's satisfaction from an infinitesimal increase in the fraction of time devoted to risky sex should compensate for the rise in the risk of contracting AIDS and suffering from its adverse implications for the duration and quality of life, for the forgone satisfaction from risk-free sex and for the discontent from the subsequent rise in the prevalence of AIDS:

$$\frac{\partial H}{\partial x} = e^{-(\rho+\mu)t} [(\alpha-1-\beta s) - 2(\alpha-1)\beta s x] + \lambda \gamma = 0. \quad (11)$$

By substituting the right-hand-side of equation (10) for  $\dot{\lambda}$  into the time-differential of equation (11) (i.e., the singular-control equation), and by recalling that  $\lambda = -e^{-(\rho+\mu)t} [(\alpha-1-\beta s) - 2(\alpha-1)\beta s x] / \gamma$ , and by rearranging terms, the no-arbitrage rule of risky sex (or the population share of the highly sexually active people under the alternative interpretation of  $x$ ) can be displayed as

$$\dot{x} = \frac{\gamma[\beta x + (\alpha-1)\beta x^2] - (\rho + \mu + \delta)[(\alpha-1-\beta s) - 2(\alpha-1)\beta s x]}{2(\alpha-1)\beta s} - \left[ \frac{\beta[1+2(\alpha-1)x]}{2(\alpha-1)\beta} \right] \frac{\dot{s}}{s}. \quad (12)$$

Substituting the right hand side of equation (6) for  $\dot{s}$ , the no-arbitrage rule can be further expressed as

$$\dot{x} = \frac{\gamma[\beta x + (\alpha-1)\beta x^2] - (\rho + \mu + \delta)[(\alpha-1-\beta s) - 2(\alpha-1)\beta s x]}{2(\alpha-1)\beta s} - \left[ \frac{\beta[1+2(\alpha-1)x](\gamma x - \delta s)}{2(\alpha-1)\beta} \right] s^{-1}. \quad (13)$$

The implications of the no-arbitrage rule for the optimal stationary levels of the representative member's risky-sex intensity and prevalence of AIDS in his, or her, group and for the possible optimal trajectories of risky-sex intensity and prevalence of AIDS off steady state are explored in the next two sections.

### III. Stationary combination of risky-sex intensity and prevalence of AIDS

By virtue of equation (6), the relationship between the stationary intensity of risky sex ( $x^*$ ) and the stationary prevalence of AIDS ( $s^*$ ) is given by the ratio of the AIDS-attrition coefficient to the AIDS-transmission coefficient

$$x^* = \frac{\delta}{\gamma} s^*. \quad (14)$$

The substitution of this relationship and the stationary condition  $\dot{x} = 0 = \dot{s}$  into the no-arbitrage rule (12) implies that the stationary share of the population infected with AIDS satisfies the quadratic form

$$\frac{1}{\gamma}(\alpha - 1)\beta\delta[2(\mu + \delta) + 3\rho]s^{*2} + (\rho + \mu + 2\delta)\beta s^* - (\alpha - 1)(\rho + \mu + \delta) = 0 \quad (15)$$

whose feasible solution is

$$s^* = \frac{-(\rho + \mu + 2\delta)\beta + \sqrt{(\rho + \mu + 2\delta)^2\beta^2 + \frac{4}{\gamma}(\alpha - 1)^2\beta\delta(\rho + \mu + \delta)[2(\mu + \delta) + 3\rho]}}{\frac{4}{\gamma}(\alpha - 1)\beta\delta[2(\mu + \delta) + 3\rho]}. \quad (16)$$

The effects of the model parameters on the stationary prevalence of AIDS indicated by equation (16) are assessed by numerical simulations. The numerical simulations start with the following choice of parameter values:  $\alpha = 1.2$  (i.e., an inducement factor of 20 percent),  $\beta = 0.5$  (i.e., a 50 percent deterioration in the quality of life after AIDS is contracted),  $\gamma = 0.5$ ,  $\delta = 0.5$  and  $\rho = 0.05$ . A value for the natural aging rate,  $\mu$ , was assigned by assuming that the probability of living to the age of 61.7 in an AIDS-free environment is equal to 0.5.<sup>13</sup> By setting  $\Phi$  in equation (4) to be equal to 0.5,  $t$  to be equal to 61.7 and  $s$  to be equal to zero (i.e., AIDS-free environment),  $\mu = (-\ln 0.5) / 61.7 = 0.011234$ . As can be seen from this expression,  $\mu$  declines with the expected lifespan in an AIDS-free environment and the probability of living beyond that age.

Substituting these parameter values into equation (16),  $s^* = 0.2304$ . By virtue of equation (14), and as identical values for  $\gamma$  and  $\delta$  are assigned,  $x^* = 0.2304$ . These results indicate that even under moderate inducement factor of 20 percent and a severe immediate deterioration in quality of life of 50 percent upon contracting AIDS, the stationary intensity of risky sex and the prevalence of AIDS are considerably high.

The effect of each of the model parameters on the stationary level of the prevalence of AIDS was simulated by changing its value, while holding the rest of the parameters at the

levels indicated above. These effects are summarised in Table 1 where each line of possible values for a specific parameter is followed by a line containing the corresponding stationary levels of the prevalence of AIDS. The column of bold entries indicates our initial choice of parameter values and the associated stationary prevalence of AIDS.

TABLE 1—SIMULATED EFFECTS OF THE MODEL'S PARAMETERS ON THE STATIONARY Prevalence OF AIDS

$\alpha$	1.025	1.05	1.10	1.15	<b>1.20</b>	1.30	1.50	1.75	2.00	2.66
$s^*$	0.0271	0.0552	0.1135	0.1726	<b>0.2304</b>	0.3372	0.5081	0.6620	0.7780	1.0000
$\beta$	0.21	0.25	0.30	0.40	<b>0.50</b>	0.60	0.70	0.80	0.90	1.00
$s^*$	0.9990	0.7601	0.5644	0.3443	<b>0.2304</b>	0.1643	0.1228	0.0952	0.0758	0.0618
$\gamma$	0.10	0.20	0.30	0.40	<b>0.50</b>	0.60	0.70	0.80	0.90	1.00
$s^*$	0.1813	0.2067	0.2186	0.2257	<b>0.2304</b>	0.2337	0.2362	0.2382	0.2397	0.2410
$\delta$	0.10	0.20	0.30	0.40	<b>0.50</b>	0.60	0.70	0.80	0.90	1.00
$s^*$	0.2869	0.2586	0.2453	0.2368	<b>0.2304</b>	0.2252	0.2207	0.2168	0.2132	0.2100
$\rho$	0.00	0.01	0.025	0.04	<b>0.05</b>	0.10	0.15	0.30	0.50	1.00
	0.2226	0.2243	0.2266	0.2289	<b>0.2304</b>	0.2372	0.2433	0.2579	0.2718	0.2923
	0.10	0.20	0.30	0.40	<b>0.50</b>	0.60	0.70	0.80	0.90	0.95
	0.2345	0.2328	0.2317	0.2310	<b>0.2304</b>	0.2299	0.2295	0.2291	0.2288	0.2287

The numerical-simulation results indicate that the stationary proportion of the group infected with AIDS largely, but concavely, rises with the inducement factor  $\alpha$  and converges to one when the satisfaction from risky sex exceeds the satisfaction from risk-free sex by 166 percent. The reduction in the prevalence of AIDS generated by a one percent improvement in the sensual quality of freely distributed condoms can be 0.855 percent when the initial satisfaction differential between risky sex and risk-free sex is 50 percent or 0.464 percent when the initial satisfaction differential is 100 percent.

Compatibly with the complacency argument, the stationary proportion of the group infected with AIDS largely increases as the risk-factor coefficient (or the rate of expected deterioration in the quality of life)  $\beta$  declines.

The stationary proportion of the group infected with AIDS slightly rises with the AIDS-transmission coefficient  $\gamma$  and slightly declines with the AIDS-attrition coefficient  $\delta$ .

The stationary proportion of the group infected with AIDS slightly rises with the rate of time preference  $\rho$ .

The stationary proportion of the group infected with AIDS declines very slightly as the probability of living beyond 61.7 years (that underlining the changes in our choice of the natural aging rate  $\mu$ ) rises. As argued earlier,  $\mu$  also declines with the expected lifespan in an AIDS-free environment. Hence, this simulation result is compatible in its direction with Philipson's and Posner's (1995) assertion of greater aversion towards risky sex in societies endowed with long life expectancy (e.g., the American society) than in societies with short life expectancy (e.g., the people of sub-Saharan Africa). However, the very small change in  $s^*$  revealed by the simulations suggests that life expectancy might not be an important factor. People who have only 20 years to live may cherish their life no less than people who have twice as many years to live.

#### IV. Phase-plane diagram of risky-sex intensity and prevalence of AIDS

The evolution of the representative member's intensity of risky sex is displayed by the no-arbitrage rule (13) and the change in the prevalence of AIDS in his, or her, group is given by the motion equation (6). In view of the complexity of equation (13), the analysis of the possible rationally optimal trajectories of risky sex and the prevalence of AIDS employs the basic set of parameters' values used in the numerical simulations of the stationary point. Consequently, the motion equations of the intensity of risky sex and the prevalence of AIDS are

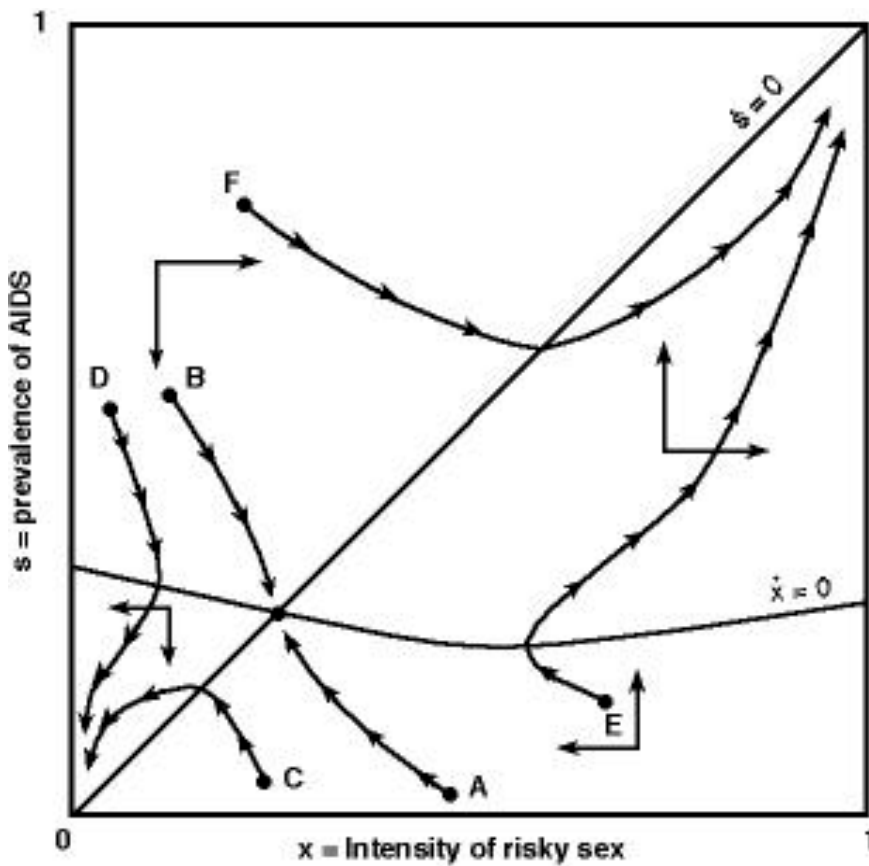
$$\dot{x} = 2.653085 + 1.061234x - (0.561234 + 0.25x^2)s^{-1} \quad (17)$$

$$\dot{s} = 0.5x - 0.5s. \quad (18)$$

By setting the left-hand-side of equation (17) to be equal to zero, the isocline  $\dot{x} = 0$  is given by

$$\left. \frac{\dot{s}}{\dot{x}} \right|_{\dot{x}=0} = \frac{0.561234 + 0.25x^2}{2.653085 + 1.061234x} \quad (19)$$

whose slope is negative for  $0 < x < 0.41461$ , equal to zero for  $x = 0.41461$ , and positive for  $x > 0.41461$  as depicted in Figure 1. Equation (17) also reveals that  $\partial \dot{x} / \partial s$  is positive. Hence,  $\dot{x}$  is positive (negative) above (below) the isocline  $\dot{x} = 0$  as displayed by the horizontal arrows in Figure 1.



**Figure 1: Phase-plane diagram of risky sex and AIDS**

By virtue of equation (6) the isocline  $\dot{s} = 0$  is given by

$$s \Big|_{x=0} = \frac{\gamma}{\delta} x. \quad (20)$$

Recalling that  $\gamma$  and  $\delta$  are set to be equal to one another, the isocline  $\dot{s} = 0$  is displayed in Figure 1 by the 45-degree line. Recalling further that  $\partial \dot{s} / \partial x = \gamma > 0$ ,  $\dot{s}$  is positive (negative) below (above) the 45-degree line as illustrated by the vertical arrows.

The stationary combination of the intensity of rational individuals' pursuance of risky sexual activity and the prevalence of AIDS in their uncoordinated group is unique and depicted by the intersection of the two isoclines in Figure 1. The horizontal and vertical arrows indicate that the stationary combination is a saddle point. Only two arms converge to this stationary point. The upper one, starting at point B, displays, despite the increase in the intensity of risky sex, a decline in the prevalence of AIDS. This is due to the high rate of attrition accompanying the initially very high prevalence of the disease. In contrast, the lower convergent arm, which starts at point A, displays, despite the decline in the intensity of risky sex, an increase in the prevalence of AIDS. This is explained by the initial high intensity of risky sex that leads to a high rate of transmission of the disease.

The rest of the possible trajectories of risky-sex intensity and prevalence of AIDS do not converge to the stationary combination. The trajectory starting at point C reveals that, despite the considerable initial intensity of risky sex, the disease will be spontaneously (i.e., without intervention) contained because of its initially low prevalence. Similarly, the trajectory starting at point D illustrates that, despite its initial high prevalence, the disease will be spontaneously contained because of the initial low intensity of risky sexual activity within the group.

However, the trajectories starting at point E and F reflect that if either the initial intensity of risky sex, or the initial prevalence of the disease, are very high, it is unlikely that the epidemic will be spontaneously contained. In these cases, as well as in cases where both the initial intensity of risky sex and prevalence of the disease are considerably high, and in the absence of effective care and prevention, the epidemic will lead to the extinction of the uncoordinated group of rational individuals.

## V. Habitual inducement and the intensity of risky sex and prevalence of AIDS

The aforementioned trajectories and the stationary combination of risky sexual activity and prevalence of AIDS were generated under the assumption that the preferences of the representative member are time-separable. An alternative approach is that of time-nonseparable preferences, where, due to habit, the inducement parameter  $\alpha$  is allowed to rise with the intensity of risky sex and to decline with the intensity of risk-free sex. That is,

$$\dot{\alpha}(t) = \alpha_1 x(t) - \alpha_2 [1 - x(t)] \quad (21)$$

where  $\alpha_1$  and  $\alpha_2$  are positive scalars. This specification displays a reinforcement effect of past behavior on the current level of inducement. Historically, unrestrained sexual activity has been dominant and even absolute, in developing regions in particular. The incorporation of the above formula of habitual inducement into the construction of lifetime portfolio of risk-free and risky sexual activities reveals that, following the introduction of sex-restraining measures to a society with no former experience, the intensity of risky sexual activity and the prevalence of AIDS will be initially larger than those suggested in our previous fixed-inducement analysis. As time progresses,  $\alpha$  and its shadow value decline, and hence it is likely that beyond a critical point in time the intensity of risky sex and the prevalence of AIDS under habitual inducement will be lower than under fixed inducement.

In the presence of habitual inducement, the representative member's decision problem is maximizing  $J$  (given by equation (8)) subject to the motion equation (6) of the prevalence of AIDS and the motion equation (21) which displays the evolution of the inducement level. The corresponding Hamiltonian is

$$H(t) = e^{-(\rho+\mu)t} [1 - \beta x(t)s(t)][1 + (\alpha - 1)x(t)] + \lambda(t)[\gamma x(t) - \delta s(t)] + \eta(t)\{\alpha_1 x(t) - \alpha_2 [1 - x(t)]\} \quad (22)$$

where  $\eta$  is the shadow value of  $\alpha$ .

In addition to the motion equations (6) and (21), the set of necessary conditions for maximum expected lifetime utility includes

$$\dot{\lambda} = -\frac{\partial H}{\partial s} = e^{-(\rho+\mu)t} [\beta x + (\alpha - 1)\beta x^2] + \lambda \delta \quad (23)$$

$$\dot{\eta} = -\frac{\partial H}{\partial \alpha} = -e^{-(\rho+\mu)t} (1 - \beta x s)x \quad (24)$$

$$\frac{\partial H}{\partial x} = e^{-(\rho+\mu)t} [(\alpha - 1 - \beta s) - 2(\alpha - 1)\beta s x] + \lambda \gamma + \eta(\alpha_1 + \alpha_2) = 0. \quad (25)$$

Rearranging the terms in the optimality condition (25) it can be shown that in the presence of habitual inducement (*HI*) the intensity of risky sex ( $x_{HI}$ ) at each instant is given by

$$x_{HI} = \frac{e^{-(\rho+\mu)t} (\alpha - 1 - \beta s) + \lambda \gamma + \eta(\alpha_1 + \alpha_2)}{2(\alpha - 1)\beta s} \quad (26)$$

whereas by rearranging the terms in the optimality condition (11) it is found that under fixed inducement (*FI*) the intensity of risky sex ( $x_{FI}$ ) is given by

$$x_{FI} = \frac{e^{-(\rho+\mu)t} (\alpha - 1 - \beta s) + \lambda \gamma}{2(\alpha - 1)\beta s}. \quad (27)$$

Thus, starting from the same level of  $\alpha$ ,  $x_{HI} - x_{FI}$  is initially positive:

$$x_{HI}(0) - x_{FI}(0) = \frac{\eta(0)(\alpha_1 + \alpha_2)}{2(\alpha - 1)\beta s(0)} > 0. \quad (28)$$

By the adjoint equation (24),  $\eta$  declines over time, and  $\alpha$  is likely to decline over time after the introduction of precautionary measures. Hence, it is possible that beyond a

critical point in time  $x_{HI} - x_{FI}$  is negative. Recalling the motion equation (6), the prevalence of AIDS is initially larger in the presence of habitual inducement than under fixed inducement, but later smaller.

## VI. Conclusion

The paper analyzed the portfolio of risky and risk-free sexual activities and the prevalence of AIDS within a group of rational people with a stochastic, dynamic optimisation model. A key feature of the model was the assumption of an inducement factor: a positive marginal instantaneous-utility differential between risky sex and risk-free sex. Another key feature was that the probability of continuing to live diminishes with the interaction between the individual's intensity of risky sex and the prevalence of AIDS in his, or her, reference group. The latter feature was interchangeably viewed as the disutility associated with a possible deterioration in quality of life whose likelihood rises with the prevalence of AIDS in the group and the individual's level of involvement in risky sex. A third feature of the model was that the prevalence of the epidemic is intensified by the intensity of risky sexual activity and moderated by attrition. The combination of these key features illustrates the complacency assertion as a decline in the risk-factor coefficient induced by the new effective drug cocktails, that led to a hike in the intensity of risky, but more satisfying, sexual activity and, subsequently, to an increase in the prevalence of AIDS beyond the level expected from the decline in the AIDS-attrition coefficient *per se*.

The model showed that along the efficient trajectory of risky and risk-free portfolio the satisfaction from an infinitesimal increase in the fraction of time devoted to risky sex should compensate for the rise in the risk of contracting AIDS and suffering from its adverse implications for the duration and quality of life, for the forgone satisfaction from risk-free sex and for the discontent from the subsequent rise in the prevalence of AIDS. The numerical-simulation results indicated that the stationary proportion of the group of rational people infected with AIDS largely rises with the inducement factor (i.e., the satisfaction differential between risky sex and risk-free sex) and, compatibly with the complacency assertion, it also largely rises as the risk-factor coefficient (or the immediate deterioration in quality of life upon contracting AIDS) declines. Even under the moderate

assumptions of a twenty percent inducement factor and fifty percent risk factor the stationary intensity of risky sex and the stationary prevalence of AIDS are considerably high—twenty-three percent. What are the policy implications of these results?

During the last twenty years a large effort has been invested in developing drugs against the AIDS virus. The existing drug cocktails do not provide a cure and immunisation. They only slow the deterioration in the quality of life and extend the life expectancy of those infected with AIDS. Using the model's terminology, they reduce, but not eliminate, the risk factor  $\beta$ . The achievements in drug development benefited only a small, rich fraction of the population suffering from the disease and, perhaps, increased, considerably in view of the numerical simulation results, the level of complacency and, subsequently, the prevalence of the epidemic in the rich countries. The policy implications are therefore clear. A greater attention should be paid to the satisfaction differential between risky sex and risk-free sex and its reduction by a widely affordable method. Much more effort should be invested in developing and widely distributing a, sensually less restraining condoms. The development and wide distribution of such high quality condoms may reduce the inducement factor and hence the intensity of risky sex and the prevalence of the epidemic in both poor and rich areas. In accordance with Table 1's simulation results, an improvement in the quality of condoms that reduces the inducement factor from 1.2 to 1.1 may lower the stationary prevalence of AIDS by almost 51 percent from 0.2304 to 0.1135. Similarly, if the present inducement factor is much greater, an improvement in the quality of condoms that reduces the inducement factor from 2.00 to 1.50 may lower the stationary prevalence of AIDS by almost 35 percent from 0.7780 to 0.5081.

The numerical simulations also revealed that the stationary proportion of the group infected with AIDS only slightly rises with the AIDS-contraction coefficient and the rate of time preference and only slightly declines with the AIDS-attrition coefficient and that it is almost insensitive to changes in the assumption about the probability of living beyond the expected lifespan in an hypothetically AIDS-free developing country.

The model also suggests that the stationary combination of risky-sex intensity and the prevalence of AIDS is a unique, saddle point. One of the two convergent arms displayed, despite the increase in the intensity of risky sex, a decline in the prevalence of AIDS

because of the high rate of attrition accompanying the initially very high prevalence of the disease. The other exhibited, despite the decline in the intensity of risky sex, an increase in the prevalence of AIDS because of the initial high intensity of risky sex that leads to a high rate of transmission of the disease. The rest of the possible trajectories of risky-sex intensity and prevalence of AIDS do not converge to the stationary combination. One prototype non-convergent trajectory revealed that, despite the considerable initial intensity of risky sex, the disease would be spontaneously contained because of its initially low prevalence. Another non-convergent prototype trajectory illustrated that the disease would be spontaneously contained, despite its initial high prevalence, when the initial intensity of risky sexual activity within the group is low. Other non-convergent trajectories reflected, however, that if either the initial intensity of risky sex, or the initial prevalence of the disease, are very high, it is unlikely that the epidemic would be spontaneously contained and, in the absence of effective care and prevention, the epidemic would lead to the extinction of the uncoordinated group of rational people.

Finally, the modification of the group-members' preferences to be time-nonseparable and the incorporation of habitual inducement into the construction of their lifetime portfolios of risk-free and risky sexual activities reveal that following the introduction of sex-restraining measures to a group of rational people with no former experience with such precautionary measures, the intensity of risky sexual activity and the prevalence of AIDS will be initially larger than those suggested by a fixed-inducement analysis. After a while it is likely, however, that the intensity of risky sex and the prevalence of AIDS under habitual inducement will be lower than under fixed inducement due to the decline of the inducement level and its shadow value.

## References

**Ahituv, Avner; Hotz, Joseph V. and Philipson, Thomas**, “The responsiveness of the Demand for Condoms to the Local Prevalence of AIDS,” *Journal of Human Resources*, 31, 4 (Fall 1996): 869-897.

**Anderson, Roy M., and Robert M. May**, *Infectious Diseases of Humans: Dynamics and Control*, Oxford: Oxford University Press, 1991.

**Baylies, Carolyn**, “Overview:HIV/AIDS in Africa: Global and Local Inequalities and responsibilities,” *Review of African Political Economy*, 27, 86 (December 2000): 487-500.

**Caldwell, John C.**, “Rethinking the African AIDS Epidemic,” *Population and Development Review*, 26, 1 (March 2000): 117-135.

**Kremer, Michael**, “Integrating Behavioral Choice into Epidemiological Models of AIDS,” *Quarterly Journal of Economics*, 111, 2 (May 1996):549-573.

**Philipson, Tomas J. and Posner, Richard A.**, *Private Choices and Public Health: The AIDS Epidemic in an Economic Perspective*, Cambridge and London: Harvard University Press, 1993.

**Philipson, Tomas J. and Posner, Richard A.**, “The Microeconomics of the AIDS Epidemic in Africa,” *Population and Development Review*, 21, 4 (December 1995): 835-848.

**Sharpe, William F.**, “Capital Asset Prices: A Theory of Market Equilibrium under Risk,” *Journal of Finance*, 19 (September 1964): 425-442.

**Thompson, Velma Montoya**, “AIDS Testing: An Economic Assessment of Evolving Public Policy,” *Economic Inquiry*, 27, 2 (April 1989): 259-269.

**Tobin, James**, “Liquidity Preference as Behavior Towards Risk,” *Review of Economics Studies*, 25 (February 1958): 65-86.

**UNAIDS**, *AIDS Epidemic Update: December 2000*, [www.unaids.org](http://www.unaids.org).

**United Nations**, *World Population Prospects: The 1998 Revision, Vol. II: Sex and Age Distribution of the World Population*, 1999.

## Endnotes

1. For a global summary of the AIDS/HIV epidemic see *AIDS Epidemic Update: December 2000*, UNAIDS, [www.unaids.org](http://www.unaids.org). For a review of the AIDS epidemic in Africa see also Baylies (2000) and Caldwell (2000).
2. Thompson (1989) has proposed that the optimal policy differs in accordance with the individual level of risk and argued against the prevalence of laissez faire as to the private-market supply of the AIDS antibody test and rules guaranteeing strict confidentiality of the test results.
3. See Tobin's (1958) and Sharpe's (1964) seminal papers.
4. Some readers may object to this portfolio approach and the possibility of non-corner solution and argue that, unlike financial asset portfolio, the risk here is to life and a rational person will not risk his, or her, life for an extra, brief, sexual thrill. The attention of the sceptics is drawn to the analogous behavior of drivers and to the following questions. Have you, presumably on the road with a rational intention of getting from A to B, never passed the speed limit or taken the more scenic, but dangerous, route? And if you happened to have a motorcycle, have you always driven it with full-body protective gear (especially on a hot summer day)?
5. Kremer (1996) has suggested that increased HIV risk may make people with high sexual activity fatalistic and hence they reduce their risky sexual activity only slightly or even increase it. This fatalistic attitude and reaction is beyond the scope of the present paper.
6. The model was constructed independently prior to reading the economic literature on the subject.
7. Ahituv, Hotz and Philipson (1996) have found that the use of condoms by youth in the United States increases significantly with the prevalence of AIDS.
8. Letting the inner integral be  $U$  and  $p(t)dt$  be  $dV$ .
9. The effect of age on the prospects of survival can be alternatively presented as a proportion of  $T - t$ . Such a representation is more compatible with the existence of an upper-bound on life expectancy than the exponential representation used in equation (4). However, it tremendously complicates the analysis.
10. Since  $s$  is the percentage of the population, rather than the number of people, infected with AIDS, it is possible to assume that  $\gamma$  is independent of  $s$ . Alternatively, the contraction of AIDS could be assumed to follow a shifting logistic process (S-curve) with a contraction rate which is increasing in the intensity of risky sex but initially increasing

and later decreasing in the prevalence of the epidemic:  $\dot{s}(t) = \gamma(x(t))s(t)[1 - s(t)] - \delta s(t)$ ,  $d\gamma/dx > 0$ . Though more realistic, this specification complicates the analysis considerably.

11. While in Anderson and May (1991) and Kremer (1996) the transmission of the disease is proportional to the rate of partner change, here it is proportional to the intensity of unsafe sex, or to the population share of the highly sexually active people under the alternative interpretation of  $x$ .

12. The time index is omitted for tractability.

13. 61.7 years is the expectation of life at birth without AIDS computed by the United Nations Population Division, United Nations (1999), with a sample of 35 African, 4 Asian and 6 Latin-American developing countries infected with AIDS.