Fundamentals  
Lecture Three

Logarithms  Simplify
\[ \log_a 18 - 2 \log_a 3 \]

Factorisation  Simplify
\[ 2x^2 - 5x - 12 \]

Algebraic Fractions  Simplify
\[ \frac{m^2 + m - 2}{m^2 - m} \]
Logarithms

Consider a simple example,

$$16 = 2^4.$$  

2 is called the base of the number, and 4 is called the index or the logarithm to base 2 of 16.

$$16 = 2^4,$$

number = base$^\text{Index}$.

OR 4 = log$_2$ 16,

logarithm = log$_{\text{base}}$ number.

More formally. If

$$N = b^x \iff x = \log_b N,$$

where $b$ and $N$ are positive real numbers.

then $x$ is called the logarithm to base $b$ of $N.$
Rules

Let $x$ and $y$ be positive real numbers.

1. $\log_b xy = \log_b x + \log_b y$

2. $\log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y$

3. $\log_b x^p = p \log_b x$

4. $\log_a a = 1$ (because $a^1 = a$)

5. $\log_a 1 = 0$ (because $a^0 = 1$)

Note: There is no rule of logarithms for simplifying either $\log (x + y)$ or $\log (x - y)$
Common Logarithms are logarithms to the base 10, and are written either as $\log_{10} N$ or, simply, $\log N$.

Natural Logarithms are logarithms to the base $e$, and are written either as $\log_e N$ or, most frequently, as $\ln N$.

Note:

(i) $\ln e = 1$.

(ii) $\ln e^x = x$,

(iii) $e^{\ln x} = x$, 
Simplify the following expressions

1. \( \log_2 16 \)
2. \( \log_5 50 + \log_5 10 - \log_5 4 \)
3. \( \frac{\log 32}{\log 8} \)
4. \( 4 \log 3 - \log 27 \)

**Exercises on Logarithms**

Exercise 1.3.2. (page 7)

For each of the questions do as many of the subquestions as you require in order to gain mastery of the basic technique.
\begin{align*}
(4) \\
(3) \\
\left(\frac{5}{3}\right) \\
(\log 3)
\end{align*}
Factorisation

*Factorisation* of an algebraic expression consists of rewriting the given algebraic expression as a *product*. The terms in the product are called *factors*.

Recall

\[ x^2 + 2xy + y^2 = (x + y)^2 \]
\[ x^2 - 2xy + y^2 = (x - y)^2 \]

Common Factors

Factorise

\[ ab + ac + 3a = \]
Common Factors by Grouping

Factorise the following expressions

1. \( ax + bx + ay + by \)
2. \( x^2 - y^2 - 6x + 6y \)
3. \( x^2 - 5x + 6 - ax + 2a \)

Difference of Two Squares

Factorise the following expressions

1. \( x^2 - y^2 \)
2. \( a^4 - b^4 \)
3. \( (2x + 3y)^2 - (x - 4y)^2 \)
\[(a + b) (x + y)\]
\[(x - y) (x + y - 6)\]
\[(x - 2) (x - 3 - a)\]

\[(x - y) (x + y)\]
\[(a - b) (a + b) (a^2 + b^2)\]
\[(3x - y) (x + 7y)\]
Quadratic Factors

Factorise the following expressions

1. $x^2 + (a + b) x + ab$

2. $x^2 - 9x + 20$

3. $6x^2 + 11x + 3$

Exercises on Factorisation

Exercise 1.4.1 (page 9)

Do as many of the questions as you require in order to gain mastery of the technique.
\[(x + a) (x + b)\]

\[(x - 5) (x - 4)\]

\[(2x + 3) (3x + 1)\]
Algebraic Fractions

Rules

1. \[ \frac{am}{bm} = \frac{a}{b} \]
2. \[ \frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c} \quad (c \neq 0) \]
3. \[ \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd} \quad (b, d \neq 0) \]
4. \[ \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \quad (b, d \neq 0) \]
5. \[ \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \quad (b, c, d \neq 0) \]

Note

1. In \[ \frac{a+m}{b+m} \] the ‘m’ cannot be cancelled.
2. \[ \frac{a}{bc} = a \div \frac{b}{c} = a \times \frac{c}{b} = \frac{ac}{b} \]
3. \[ \frac{a}{bc} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc} \]
Examples

Simplify the following expressions

1. \[
\frac{(x-2)}{(x+2)} \times \frac{3x+6}{x^2+2x-8}
\]
2. \[
\frac{2}{a-3} \div \frac{6}{a^2-9}
\]
3. \[
\frac{3}{b^2-4} - \frac{5}{3b-6}
\]
\[ \left( \frac{3}{x+4} \right) \]

\[ \left( \frac{a+3}{3} \right) \]

\[ \left( \frac{-5b-1}{3(b-2)(b+2)} \right) \]
Exercises on Algebraic Expressions

Exercise 1.5.2 (page 11)

Do as many of the questions as you require in order to gain mastery of the technique.