Fundamentals Lecture Two

Indices Simplify
\[6x^3y^{-2} \times \frac{1}{24}x^{-5}y^4\]

Surds Simplify

1. \[\frac{\sqrt{5}}{\sqrt{45}}\]

2. \[\frac{5}{1+\sqrt{5}}\]
Definition

If $a$ is a real number and $n$ is a natural number, then

$$a^n = a \times a \times \ldots \times a \text{  (n  factors)}.$$  

$n$ is called either the index of $a$, the exponent on $a$, or the power to which $a$ is raised. $a$ is called the base. □

Rules

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(ab)^m = a^m b^m$
4. $(a^m)^n = a^{mn}$
5. $a^{-m} = \frac{1}{a^m}$
6. $a^0 = 1$
7. $a^{m/n} = (\sqrt[n]{a})^m = n \sqrt{a^m}$
Examples

Simplify the following expressions

1. \[
\frac{3xy^2 \times 4x^3 \times y}{2xy \times 4y^2}
\]

2. \[
\frac{6x^{-4} \times 2x^3}{3x^{-3}}
\]

3. \[
(5x^2y^{-3/2}z^{1/4})^2 \times (4x^4y^2z)^{-1/2}
\]
\[ \left( \frac{3}{2}x^3 \right) \]

\[ (4x^2) \]

\[ \left( \frac{25}{2}x^2y^{-4} \right) \]
Definition

A *surd* is the name given to an *irrational* number that can be expressed in the form $n\sqrt{a}$.

(See definition of irrational p1-1)

**Notation**

$\sqrt{a}$, $\sqrt{a}$ or $a^{1/2}$ means “square root of $a$”

$n\sqrt{a}$, $n\sqrt{a}$ or $a^{1/n}$ means “$n^{th}$ root of $a$”

Irrational numbers such as $\sqrt{2}$, $5\sqrt{3}$, $\sqrt[3]{7}$… are *surds*, whereas $\sqrt{4}$, $\sqrt[3]{27}$, $3\sqrt{9}$… are *not*, since they can be evaluated *exactly*.

**Question.** Evaluate exactly $\sqrt{4}$, $\sqrt[3]{27}$ & $3\sqrt{9}$. 
Rules

1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$

2. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

3. $(\sqrt{x})^2 = x$ provided $x \geq 0$

4. $\sqrt{x^2} = \begin{cases} +x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$

5. $a\sqrt{x} \pm b\sqrt{x} = (a \pm b)\sqrt{x}$
Examples

Simplify the following expressions

1. $4\sqrt{3} + 5\sqrt{7} - 2\sqrt{3} + \sqrt{7}$

2. $2\sqrt{3} \times 4\sqrt{5}$

3. $\sqrt{18} \div \sqrt{2}$

4. $6\sqrt{15} \div 2\sqrt{3}$

5. $\sqrt{8}$

6. $\sqrt{3} \times \sqrt{21}$

7. $5\sqrt{18}$

8. $\sqrt{20} + \sqrt{45}$

9. $\sqrt{12} - \sqrt{27} + \sqrt{75} - \sqrt{15}$
\[(2\sqrt{3} + 6\sqrt{7})\]
\[(8\sqrt{15})\]
\[(\pm 3)\]
\[(3\sqrt{5})\]
\[(2\sqrt{2})\]
\[(3\sqrt{7})\]
\[(15\sqrt{2})\]
\[(5\sqrt{5})\]
\[(\sqrt{3} \left[ 4 - \sqrt{5} \right])\]
Rationalising the Denominator

The *conjugate* of $\sqrt{a} + \sqrt{b}$ is $\sqrt{a} - \sqrt{b}$. Multiplying a surd by its conjugate always gives a rational number

\[
(\sqrt{a} + \sqrt{b}) (\sqrt{a} - \sqrt{b})
\]

\[
= (\sqrt{a})^2 - (\sqrt{b})^2,
\]

\[
= a - b.
\]

Using this rule we can express any surd with a rational denominator.
Examples

Simplify the following expressions.

1. \( \frac{1}{\sqrt{a}} \)

2. \( \frac{1}{\sqrt{a} + \sqrt{b}} \)

3. \( \frac{3}{\sqrt{5}} \)

4. \( \frac{2}{6 - \sqrt{3}} \)

5. \( \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \)
\[
\left( \frac{\sqrt{a}}{a} \right)
\]

\[
\left( \frac{\sqrt{a} - \sqrt{b}}{a - b} \right)
\]

\[
\left( \frac{3\sqrt{5}}{5} \right)
\]

\[
\left( \frac{12 + 2\sqrt{3}}{33} \right)
\]

\[
(5 + 2\sqrt{6})
\]
Equality of surds

If \( a + \sqrt{b} = c + \sqrt{d} \) where \( a \) and \( c \) are rational and \( \sqrt{b} \) and \( \sqrt{d} \) are surds, then \( a = c \) and \( b = d \).

Examples

Find the values of \( \alpha \) and \( \beta \) in the following expressions

1. \( \alpha + \sqrt{\beta} = 2 + \sqrt{5} \)

2. \( 2\alpha + 3\sqrt{\beta} = 4 + 3\sqrt{2} \)

3. \( \alpha \sqrt{3} = \sqrt{27} \)

4. \( \alpha + 2\sqrt{\beta} = (\sqrt{3} + \sqrt{5})^2 \)
$$(\alpha = 2, \beta = 5)$$

$$(\alpha = 2, \beta = 2)$$

$$(\alpha = 3)$$

$$(\alpha = 8, \beta = 15)$$
Exercises on Indices and Surds

Exercise 1.2.4 (pages 4& 5)

For each of the questions do as many of the subquestions as you require in order to gain mastery of the basic technique.

DO NOT do question 4 on the surds worksheet.