THE UNIVERSITY OF WOLLONGONG

SCHOOL OF MATHEMATICS AND APPLIED STATISTICS

MATH111: Applied Mathematical Modelling 1

Skills Test: Difference equations

Time allowed : 40 minutes

Number of questions: 15 multiple choice

Attempt all questions.

CALCULATORS

ARE PERMITTED.

Please Note: Examination Paper is printed on both sides.
1. Consider the difference equation $a_{n+1} = -3a_n + 5$ with the initial condition $a_0 = 1$. What is $a_3$?
   
   (a) $a_3 = 8$.
   (b) $a_3 = -19$.
   (c) $a_3 = -1$.
   (d) $a_3 = 2$.

2. Which difference equation represents the sequence 1, 3, 6, 10, 15, 21, ...?
   
   (a) $a_n = a_{n-1} + 2$
   (b) $a_n = a_{n-1} + 3$
   (c) $a_n = a_{n-1} + n - 1$
   (d) None of the above

3. You open a bank account with $500. You add $25 to your account each month, and the bank pays you 2.4% interest compounded monthly. Which difference equation describes how your account balance changes from month to month?
   
   (a) $a_{n+1} = 0.2a_n + 25 + 500$ with $a_0 = 0$
   (b) $a_{n+1} = 0.002a_n + 25$ with $a_0 = 500$
   (c) $a_{n+1} = 1.002a_n + 500$ with $a_0 = 25$
   (d) $a_{n+1} = 1.002a_n + 25$ with $a_0 = 500$
   (e) None of the above

4. Suppose that $d_{n+1} = 0.9d_n + 2$. What is the steady-state value?
   
   (a) 9
   (b) 2
   (c) 20
   (d) 20/9
   (e) None of the above
5. The following difference equation describes the value of a car, where \( n \) is in years.

\[ V_{n+1} = 0.86V_n. \]

Which of the following is a true statement?

(a) The value of the car increases 86% each year.
(b) The value of the car decreases 86% each year.
(c) The value of the car decreases by 14% each year.
(d) The value of the car will eventually level out at $8600.
(e) None of the above.

6. The difference equation \( a_{n+1} = 1.08a_n \) might model the population of some species. If \( a_0 = 5000 \), which of the following statements is true?

(a) \( a_{10} > 5000 \)
(b) \( a_{40} < 5000 \)
(c) It is possible that \( a_{30} = 5000 \)
(d) More than one of statements (a–c) could be true
(e) None of statements (a–c) has to be true

7. We want to test the solution \( a_n = 6n + C \) in the difference equation \( a_{n+1} = a_n + 6 \). What equation results from substituting the solution into the difference equation?

(a) \( 6n + 1 + C = 6n + C + 6 \)
(b) \( 6(n + 1) + C = 6n + C + 6 \)
(c) \( 6(n + 1) + C(n + 1) = 6n + Cn + 6 \)
(d) \( a_n + 6 = 6(a_n + 6) + C \)
(e) None of the above

8. \( a_n = 3^n \cdot 5 + 2 \) is a solution to \( a_{n+1} = 3a_n - 4 \) with \( a_0 = 2 \).

(a) True
(b) False
9. The equation $a_{n+1} = na_n + a_n a_{n-1} + n^3$ is nonlinear. Which terms makes it nonlinear?

(a) $na_n$
(b) $a_n a_{n-1}$
(c) $n^3$
(d) All of the above

10. The solution to $a_{n+1} = 5.2a_n$ with $a_0 = 6$ is

(a) $a_n = 6^n (5.2)$
(b) $a_n = 6^n C$
(c) $a_n = 5.2^n (6)$
(d) $a_n = 5.2n + 6$
(e) $a_n = 5.2^n (3)$

11. Which best describes the long-term behaviour of the solution $a_n = (-2)^n$ (12)?

(a) As $n$ gets large, the values of $a_n$ grow without bound.
(b) As $n$ gets large, the values of $a_n$ decrease without bound.
(c) As $n$ gets large, the values of $a_n$ oscillate, getting farther and farther away from zero.
(d) As $n$ gets large, the values of $a_n$ oscillate, getting closer and closer to zero.
(e) None of the above.

12. The solution to $a_{n+1} = 4a_n + 3$ with $a_0 = 5$ is

(a) $a_n = 4^n (-1) - 1$
(b) $a_n = 4^n (5) - 1$
(c) $a_n = 4^n (4) - 1$
(d) $a_n = 4^n (6) - 1$
(e) None of the above
13. Discuss the long-term behaviour of the solution $a_n = 0.7^nC + 2$ with $a_0 = 1$.

(a) This solution will increase, converging to the steady-state value of 2.

(b) This solution will decreases forever.

(c) This solution will increase forever.

(d) None of the above.

14. The solution to a difference equation is $a_n = \frac{17}{2} + \frac{5}{2}2^n$. What was the initial condition, $a_0$?

(a) $\frac{17}{2}$

(b) $\frac{5}{2}$

(c) 2

(d) 11

15. Upon graduation from university you land a job with a starting salary of $35,000. You are told that as long as your performance is up to par, you can expect a 3% raise each year. Additionally, at the end of your first year you will receive a bonus of $100, and at the end of each year after that you will receive a bonus equal to $100 times the number of years you have completed. On the first day of your job, you open a bank account with $200. This account will earn interest at a rate of 5% compounded annually, and you decide that each year you will deposit your bonus into the account. If $a_{n-1}$ represents the amount of money after $n - 1$ years, which of the following difference equations models your account balances?

(a) $a_n = 1.03a_{n-1} + 100 + 35,000$

(b) $a_n = 1.05a_{n-1} + 100$

(c) $a_n = 1.05a_{n-1} + 100n$

(d) $a_n = 1.05a_{n-1} + 1.05 (100) (n - 1)$

(e) $a_n = 1.03a_{n-1} + 1.05 (100) + 200$

(f) None of the above
MATH111: Applied Mathematical Modelling 1
Spring Session 2007
2007 Skills Test A

Student Name: ___________________________ Student Number: ________

Answer Sheet

Completely fill in the appropriate box for each question: [ ]

1. [A] [B] [C] [D] [E] [F]
2. [A] [B] [C] [D] [E] [F]
3. [A] [B] [C] [D] [E] [F]
4. [A] [B] [C] [D] [E] [F]
5. [A] [B] [C] [D] [E] [F]
6. [A] [B] [C] [D] [E] [F]
7. [A] [B] [C] [D] [E] [F]
8. [A] [B] [C] [D] [E] [F]
9. [A] [B] [C] [D] [E] [F]
10. [A] [B] [C] [D] [E] [F]
11. [A] [B] [C] [D] [E] [F]
12. [A] [B] [C] [D] [E] [F]
13. [A] [B] [C] [D] [E] [F]
14. [A] [B] [C] [D] [E] [F]
15. [A] [B] [C] [D] [E] [F]
MATH111: Applied Mathematical Modelling 1
Spring Session 2007
2007 Skills Test A

Student Name: Correct Answers: Student Number:__________

Answer Sheet

Completely fill in the appropriate box for each question:

1. A   B   C   D   E   F

2. B   C   D   E   F

3. C   D   E   F   G

4. C   D   E   F   G

5. C   D   E   F   G

6. A   B   C   D   E

7. B   C   D   E   F

8. B   C   D   E   F

9. B   C   D   E   F

10. C   D   E   F   G

11. C   D   E   F   G

12. D   E   F   G   H

13. A   B   C   D   E

14. D   E   F   G   H

15. C   D   E   F   G

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